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A Study on the Investigation of Students' Reasoning Skills in Secondary Geometry Proofing

Wei Liu[1], Huda Affas[2], David Osmond[3]

[1,2,3] Hillcrest Christian College, Crest Education, Australia

ABSTRACT: Geometry reasoning is an ability that helps students build their thinking skills in logic structures, deductive reasoning, analytical reasoning, and problem-solving. Geometric proof is a key indicator to develop those skills. However, writing proofs is challenging for students, especially those unwilling to work hard. This case study used a mixed method to explore Year 10 students' geometry reasoning abilities in proving geometric questions and aims to find effective strategies for teaching proof. Data were collected through tests, demonstrated in percentages, and van Hiele's model rubric was applied to analyse students' geometry reasoning levels. Four examples representing different reasoning levels were examined, discussed and combined with the interviewed data from the 10 participants, strategies that targeted students' needs were summarised for future planning and teaching. Qualitative data were categorised and themed to provide details for teaching and learning. The study's results showed that about 88 % of students belonged to levels III and IV, while, about 12% of students, whose level belonged to Levels I and II needed support in writing proof. The results of this study suggest that effective teaching methods and strategies are required to support students' geometry reasoning abilities.

KEYWORDS: geometry reasoning, geometry proofing, geometry learning, mathematical ability

INTRODUCTION

Geometry is an important branch of mathematics. It occupies a particular place within mathematics; it appears as a model of physical space, and it follows that the objects it deals with (e.g. lines, planes, points) are directly taken from sensory experience, unlike in the rest of the areas of mathematics. Due to geometry diversity, it can serve as a source of creative activity at different levels and a place to train various reasoning skills. Junior high school students in Grades seven or eight have gradually begun to develop from concrete image thinking to abstract logical thinking. Although this logical thinking is tendentious and empirical, the independence of thinking has been revealed ([1]; [2]). Secondary school students are expected to be able to evaluate conjectures and assertions, to reason deductively and inductively by formulating mathematical arguments, and to develop and maintain their reasoning skills ([3]; [4]; [5]). If reasoning skills are not fully developed, students will see mathematics as a collection of specific rules and boring calculations. The reasoning skills students demonstrated whilst problem-solving will allow us to observe how the students associate geometric concepts and by what means they reach a solution.

LITERATURE REVIEW

Geometry is one of the most difficult topics in mathematics, as it requires a high level of thinking ([1]). Geometric reasoning skills are strongly required in solving geometric problems because when studying geometry, students need to investigate, measure, visualise, compare, change, and classify objects ([6]).

Reasoning skills in geometry

When it comes to learning geometry, visualisation, reasoning, and spatial ability play a significant role ([7]). Visualisation is a skill that helps students recognise shapes, create new shapes or objects, and reveal relationships between them ([8]; [9]). The processes of visualisation and reasoning are parts of mathematical thinking ([10]; [11]) while geometric thinking involves the cognitive processes of visualisation and reasoning ([12]; [13]), and these cognitive processes are interconnected, promoting students' success in geometry. Battista ([14]) said that geometric reasoning refers to the act of "inventing and using formal conceptual systems to investigate shape and space" (p.843). ([2]; [15]) emphasised in their study that knowledge, visual perception and logical arguments are three factors that affect students' responses in geometry learning.

Studies suggest that different methods and techniques are necessary for students to develop reasoning skills in geometry. For example, Sahin and Kendir ([16]), and Altan and Temel ([17]), described metacognition-based education as having led to the development of mathematical reasoning and Maqsud (1998, cited by [18]) found that metacognitive strategies influenced low-achieving South African students. Cooperative learning should be used to improve students' mathematical reasoning ([6]; Kramarski et al, 2001 cited by [19]), and communication skills are important for the development of students' reasoning skills ([20]; [21]). Many researchers claimed that visualisation could be improved by training and by using materials ([22]) or through instruction methods ([23]).

There are some studies focused on different reasoning categories in learning geometry, such as spatial reasoning ([24]), adaptive reasoning skills ([25]) and reasoning skills in writing proof ([26]) and quadrilaterals ([27]) which provided some theories analysis about reasoning in geometry. Varieties of methods of how to improve students' geometry reasoning skills were explored by research such as using interactive software ([28]), hands-on activities ([29]) and solid models ([30]) etc.

There are many studies about reasoning skills in geometry, such as comparing students' reasoning abilities from different schools' different grade levels, or genders ([31]), and analyzing the situation of students' reasoning skills ([26]; [32]; [30]). In China, most studies are mainly focused on analyzing the situation of students' reasoning skills ([33];[34]; [35]). The gap is that there are few studies about models that teach or support students in learning and developing reasoning in geometry. In addition, studies show that good mathematical reasoning skills are imperative to proof-writing performance ([36]; [37]; [38]). Duval ([39]) pointed out that to determine the difficulties students encounter in geometry, it is necessary to identify the cognitive processes that underlie geometric processes. Based on this reason, to better understand students' reasoning skills in geometry, it is important to investigate students' levels of geometric reasoning and find out the factors that influence students' learning, which will benefit future teaching and learning.

Geometric proof

Geometric proof is considered a high level of mathematical reasoning ([40]) because it involves gathering evidence ([41]). The proof process manifests mathematical reasoning tasks ([42]). In carrying out the geometric proving process, different kinds of difficulties cause students to be unable to complete the proof. In an attempt to investigate how students learn and solve geometric proofs, relative aspects such as teaching proof techniques; verifying the evidence; demonstrating the evidence; the difficulty of transitioning from high school mathematics to undergraduate mathematics and the importance of proof in educational settings ([41]; [43]; [13]) were explored. "Verify, explain, communicate, and systematize statements into deductive systems" ([44], p. 2) are words that are usually used in proof to attempt to understand issues, if students understand how to solve problems, they can provide descriptions and solutions ([45]), however, when they are not confident in the contents proof processes or reasoning skills, they will stop trying to solve the problem. Some issues are related to common errors or misconceptions. Sometimes, to prove the conclusion, students tend to assume the conclusion exists ([13]). Some issues are due to students having relatively low geometry knowledge ([46]), and struggling with remembering formulas and arranging the steps in solving problems in the geometry domain ([47]).

There are many reasons why teaching proves unsuccessful. While doing proving, students overlook the need for evidence because the questions are obvious to prove ([48]). Students are unable to distinguish between analysing, explanation or proof. Students face challenges when writing proofs and need help to convert their thinking to provide valid proofs, not just plausible ones. Some research may indicate positive ways forward. A range of work, such as that by De Villiers ([49]) and Hanna ([50]), suggests that increasing the emphasis on one of the major functions of proof, that of explanation, is central to learners' success in learning to prove. Explaining a higher profile, it is claimed, should help teachers connect with students' reasoning and guard against the students experiencing learning to prove as no more than a ritual determined by the teacher. However, the mathematical proof is more structurally specific than a general explanation. In particular, learning to prove involves learners "taking on this precise form of reasoning as their own" ([51], p.236) such that they tend to require proof-like explanations to become convinced. In addition, the availability of new tools, especially computer tools such as dynamic geometry software, also has implications for how proof and proving can be taught and learnt. Miyazaki, Fujita and Jones ([52];[53]) proposed a theoretical framework for the structural understanding of proof, which stated that this low-chart proof can scaffold junior secondary students' understanding of the structure of writing proof. A new tool such as the Learning Trajectory of Geometric Proofs (LTGP) ([43]) provides an opportunity to study how students' understanding of proof structure changes over time, it aims to help teachers to follow the track to provide support in teaching the structures of geometry proof.

Proofs are also encountered in other parts of the mathematics curriculum ([54]). In the current version of the Victorian Curriculum (Victorian Curriculum and Assessment Authority (VCAA), [55]), geometric reasoning skills include "formulate proofs involving congruent triangles and angles properties", and "apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes". From the Curriculum, students are to be taught to distinguish between practical demonstrations and proofs and to show step-by-step deduction in solving a geometrical problem, they are expected to obtain the ability to communicate a proof using a sequence of logically connected statements ([56]; [55]). Proving geometry problems requires reasoning skills and understanding geometric concepts and evidence processes. Therefore, to solve such problems, effective teaching methods and learning methods are needed to help students master proof skills, and students' levels need to be tested so that they are familiar with relevant content at certain stages of their personal geometry learning process and the support content is matched to their level. It can be summarised that geometric proofs are affected by many factors, not only from the course content but also by students' reasoning ability, critical thinking ability and other cognitive aspects during the learning process. In addition, the challenge of teaching this curriculum is to develop teaching methods that neither bore students nor force them to simply memorize the material. This will certainly require teaching methods that may involve technology such as dynamic geometry, as well as discourse participation methods and assessment methods that reduce the pressure of rote memorization ([48]).

Aim of the study

The purpose of this study is to explore students' reasoning levels, find out factors that influence students' reasoning ability in geometry, how to support students' learning and develop reasoning skills, further explore how to develop students' reasoning skills in geometry proof and highlight pedagogical planning and teaching in geometry.

Three research questions underpinning this study are: Students' level of reasoning skills and their performance of geometric proofs; What kind of difficulties are faced by the students in solving geometry problems using reasoning? How do teaching strategies address weaknesses in secondary students' geometric reasoning skills?

METHOLOGY

Research design

The method used in this research is qualitative research with a case study design which aims to determine the mathematical reasoning abilities of high school students on geometry proof. The van Hiele model of geometric as a rubric was applied when analysing students' geometric reasoning skills. The van Hiele model of geometric thought has served to characterize individuals thinking in geometry for more than 40 years now, it consists of four levels, that is, in the development of students' geometric reasoning including visual reasoning, descriptive reasoning, analytic reasoning, relational-inferential property-based reasoning, and formal deductive proof ([57]). These levels ranged from the lowest to the highest - Level 0-visualization, Level 1-analysis, Level 2-informal deduction, Level 3-formal deduction and Level 4-rigor ([58], p.31). The descriptors of each of these levels are presented in Table 1.

Table I revised from Mayberry, ([58]) p.59

Level I	*Level II*	*Level III*	*Level IV*
Students perceive properties isolatedly and unrelatedly. Students do not notice the relationship between properties or perceive the relationships between different figures.	Students perceived the relationships between properties and between figures. Students understand the logical implications and class inclusions Students do not understand deduction.	Students perceive properties Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof	Students understand the formal aspects of deduction. Students should understand the role and necessity of indirect proof and proof by contrapositive

The research instruments included a geometry test and an interview sheet. The research instrument was validated by three mathematics teachers who each have more than 15 years of teaching experience. Reasoning ability data were collected through tests and semi-structured interviews. The data analysis phase included: (1) providing the test items to the students; (2) analyzing the test results; (3) interviewing several students; and (4) analyzing the interview results. The last step was to conclude.

Participants

This study took 60 Grade 10 students who study General Mathematics at a Christian College, located southeast of Melbourne, Australia. According to the students' reasoning ability, they were divided into multiple levels (see Table 1), and 10 students were selected from each group to participate in the interview.

Data analysis

The research data included the results of the geometry proof test, which were presented as percentages, the data were categorized using references adapted from other research ([7]; [21]) into 6 groups (shown in Table 1). Four examples were analysed and discussed. Students' work is measured using the van Hiele model and determined by three mathematics teachers and researchers. Reasoning skills that students are expected to demonstrate include logical statements in writing, diagrams, and proof structures. Data from the interview were analysed and themed (shown in Table 2), which provided an opportunity to find out more patterns in students' reasoning process when doing the proof questions ([30]), interview data offered further details about future pedagogical planning.

RESULTS

60 students participated in the test, and 10 students participated in the semi-interview. The data were categorized using references adapted from other researchers ([59]; [7]), and combined with the rubric, given the category-appropriate reasoning skills.

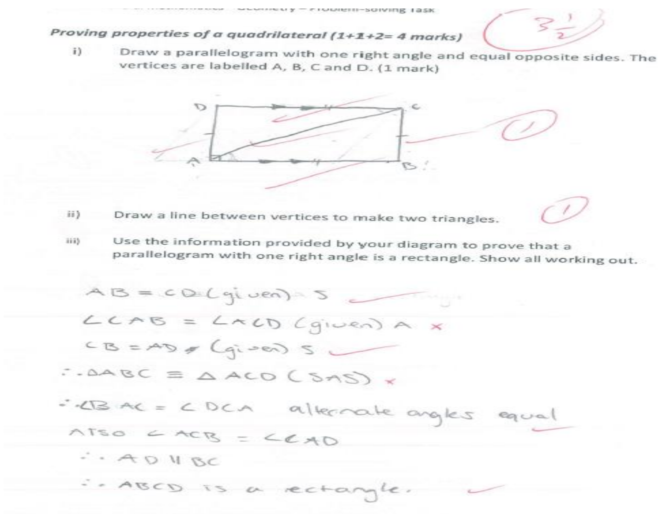
Table 1 the score of Year 10 General Math students' geometry reasoning ability

Score	***Category***	***Frequency***	***Percentage***	***level***
86-100	excellent	16	26.67%	Level IV
76-85	Very good	27	45.00%	Level III &IV
61-75	good	10	16.67%	Level III
51-60	satisfactory	2	3.33%	Level II
41-50	approaching	3	5.00%	Level II
0-40	Under standard	2	3.33%	Level I

11.66% of students belong to levels I and II, which will be the main group that teachers would like to target and facilitate the improvement. Analysing examples of different levels of student's work out.

Results from students test

Example A



Level IV

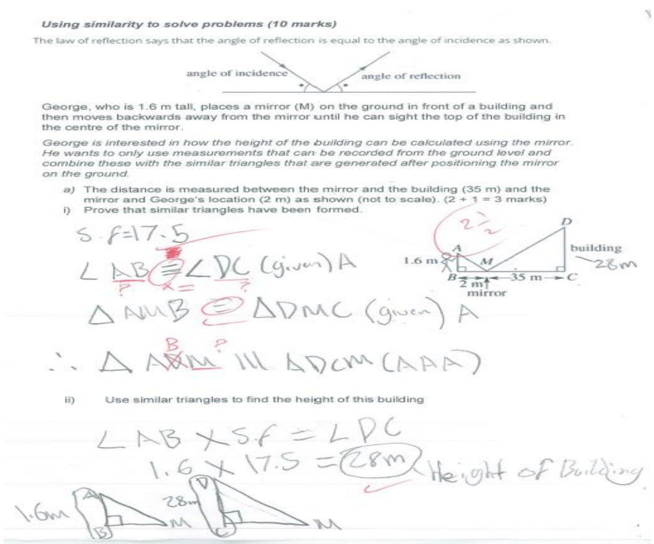
Student understood the correct geometric symbols.

They were able to use congruence in proof. However, they made a mistake in choosing the right congruency test.

They were able to write the congruency statement mostly accurately.

They were able to test for a type of quadrilateral.

Example B



Level III

The student understood the concept of scale factor.

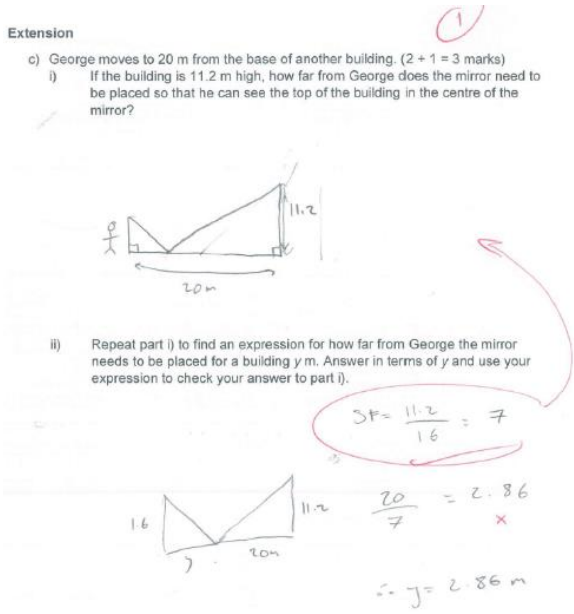
They were able to calculate and apply a scale factor in similar figures.

They were able to write mathematical statements showing an understanding of equivalence in geometric reasoning.

They were able to interpret the accumulation of equivalence to demonstrate similarity but with errors.

They were able to identify the appropriate test for similarity.

Example C



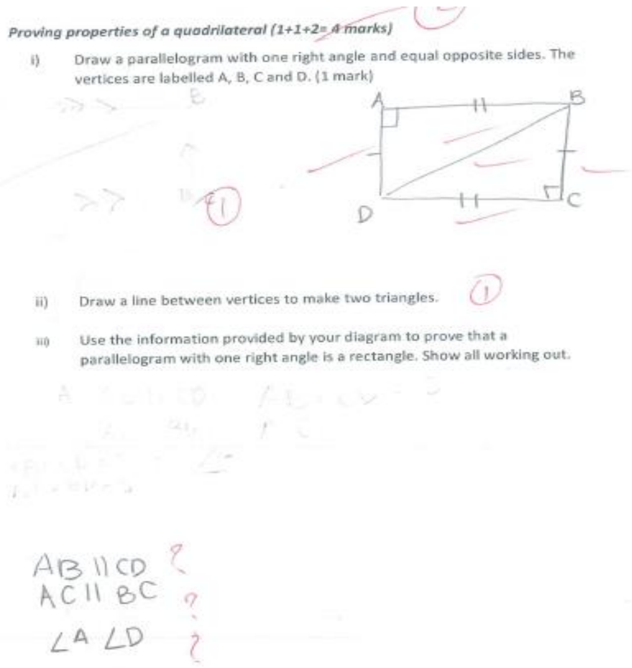
Level II

Students could use grid paper to sketch the diagram and highlight the matching sides or angles to ensure they are equal.

Students mixed the corresponding sides or angles when writing similarity or congruence proof steps.

Students did not provide a correct formula for the scale factor.

Example D



Level 1

The student can sketch the diagram based on the statement of the question.

The student understood the geometric properties of the quadrilateral.

The student was unable to show skills in writing geometric proofs.

Interview data

Table 2 data from the interview

Category	*Board understanding*	*Themed phrases (for teachers and students)*
understanding	Literacy problems with lower-level students The other level students demonstrate a good understanding of the worded problems. Geometry symbols and terminology	Understanding Curriculum and Planning
processes	Structures of proof	Teaching
Resources	CAS , scientific calculator Revision worksheets, practice tests, videos, Essential assessment, Education Perfect	Teaching and learning
Teaching & learning	Demonstrate examples with diagrams Proof structures Work examples with emphasis on important processes and steps Order is important when writing the similarity test. Feedback Summary Notes	Teaching and learning

DISCUSSION

When asked about their understanding of geometry reasoning skills, 80% of the participants in the interview chose ‘Geometry knowledge’, the second higher percentage was ‘Critical understanding of the question’ and ‘Problem-solving ’, which was both 60%, ‘logical argument’ which was 50%, following is ‘Analyzing assumptions and relationships’ which was 30%, no one chose ‘Spatial reasoning’, which is reasonable as the geometry test did not include spatial questions, as this section does not cover in the test, students do not familiar with the terminology, also, this terminology is senior curriculum approach. From the interview, data showed that most students have a very good understanding of geometry reasoning skills and can identify which questions related to those skills. However, for students at the below-standard level, word problems are challenging.

Based on the four examples, teaching strategies were discussed to provide further support for students who made similar mistakes.

Teaching strategies for example A (Figure 1), such as providing practice questions showing the structure of proof but requiring students to fill in missing information. Questions should progress to decreasing levels of scaffolding. Example B (Figure 2), students need support such as providing step-by-step instructions; breaking down and simplifying questions for easier understanding; and providing consistent practice with related hands-on activities, worksheets and puzzles. Example C (Figure 3), shapes embedded in a diagram can be difficult for students to identify, strategies such as colouring the shape within a diagram and then re-drawing it beside the original shape can provide visualise length and angle relationships between those two shapes. Example D (Figure 4) showed that level I students need more study on exploring quadrilaterals, there are six special quadrilaterals identified in the Australian Curriculum: squares, rectangles, parallelograms, rhombuses, trapeziums and kites. Students demonstrated a basic understanding of the properties of parallelograms, but the diagram is not sketched properly, students did not show an understanding of the difference between rectangles and parallelograms, notifying that the right angles, the width is perpendicular to the length is the main difference feature to identify rectangle and parallelograms, further get the conclusion that rectangles are special parallelograms. Teaching strategies include 1) providing students opportunities to see the properties in various situations; 2) practising sketching geometry diagrams and using appropriate symbols and languages to describe the shapes; and 3) using physical models. 4) modelling how to write the proof, address when the order of the ideas is important and explain the reason. For example, in examples A and B, the order of the angles does not matter, however, in the SAS test, the angle should be stated between the two sides to show that those two sides produce the angles.

Geometric reasoning requires studying objects and writing proofs of those objects. It is abstract, so learning involves a process of thinking or reasoning ([60]), and appropriate learning strategies are needed during teaching ([61]). Following the analysis of the results, teaching strategies are discussed to help students and make them more confident in writing the proof structures: 1) Teaching demonstration including using geometric language. Addressing some terminologies that students are confused with, providing separate examples, congruence and similarities, test rules are very similar. 2) Interpret the written questions and convert them into a labelled diagram to provide a visual presentation. Discuss the geometric characteristics, prior knowledge, and skills that can be applied to solve the problem. 3) Written structures of proof. Students identify which similarity or congruence test they can use in writing the proof and providing a correct test for their conclusion. Ask students to provide a reasoning explanation for the structure. 4) Encourage students to question whether the answer makes sense. Using their solution to test other related questions.

CONCLUSION

Based on the results and discussion, it could be concluded in this study that students can identify a shape based on its features and properties. The encouraging finding in this study is that 88.3% of students at levels III and IV, means that students can organise their knowledge towards a higher level of geometrical thinking, and they can write the proof logically and reasonably, their understanding of the structure of evidence and their proof construction abilities are reasonable. Another finding provides a perspective of viewing the difficulties in writing a geometric proof, such as paying attention to students who are at levels I or II, thus, an effective teaching approach must be implemented to facilitate students reasoning development. Based on the test results and interview data, teachers and the researchers are fully aware of the purpose of the test and research, they summarised an approach to teaching students to write proof logically and instructed, such as ideas on the diagram, writing down reasons, writing statement to be proven, as well as different types of reasoning and methods of proof. A learning environment that cultivates the culture of reasoning will be helpful, towards learning as a logical and structural process of geometry knowledge transfer, viewing geometric reasoning skills as a very formal communication, and encouraging the whole classroom to practice, will make it possible for students' geometric reasoning skills run well and can improve their level of reasoning.

A few factors limited this study. First, more tests need to be done to analyse students' geometric reasoning levels. Second, other rubrics can be applied to categorise students' reasoning levels. Third, more participants getting involved in the interview will provide information and details about how students reflect on the test and their understanding. All these factors could contribute to further examination of the validity of van Hiele's rubric.

Future research might include further confirmation of the validity of van Hiele's rubric, and how to use the rubric if students are between two levels, then teachers need to determine students' levels. Students' attitudes and beliefs towards geometric reasoning. The pedagogy approaches proposed in this article should be implemented in the future to help students make progress, and these strategies should be evaluated.

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