

## Hypothetical Learning Trajectory (HLT): Analisis Bibliometrik Berbasis Vos Viewer Bibliometric



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**ABSTRACT:** The purpose of this bibliometric analysis is to see the opportunities and development of research trends related to Hypothetical Learning Trajectory (HLT) from 2019 to 2024. Through bibliometric approach, this study will map publication trends, based on Scopus database (2019-2024) by using Vos Viewer to analyze the data. The results of this study showed the most publications and citations in 2022 as many as 41 publications and 445 citations, and a decrease in publications and citations from 2023 to 2024. Terms often associated with Hypothetical Learning Trajectory i.e., students, concepts, mathematics.

### INTRODUCTION

Concept ability becomes an essential competence in mathematics education, which is expected to analyze and evaluate information independently. Concepts are not only important for mathematical problem solving but also for effective decision making in everyday life (Halpern, 2020). So that this learning approach can be made as one way to improve the necessary skills.

One innovative approach that draws attention in the context of mathematics learning is *Hypothetical Learning Trajectories* (HLT). HLT offers a framework for designing learning experiences that focus on developing students' understanding through active interaction with mathematical concepts (Simon, 2019). By integrating HLT in learning, students are not only invited to understand the material but also to concepts regarding the relationship between concepts and their application in real situations (Watson & Chick, 2020).

The importance of HLT-based mathematics learning focuses on engaging students in the learning process, encouraging them to conceptualize and be creative. Leatham and Barlow (2020) state that HLT not only serves as a guide in planning learning, but also as an evaluation tool that allows teachers to assess the effectiveness and progress of students. This shows how important HLT is in improving the quality of education, especially in mathematics learning.

In the context of *mathematics learning Hypothetical Learning Trajectory* (HLT) becomes one of the fundamental frameworks for designing effective learning experiences. HLT refers to a lesson plan that describes the student's learning journey, including learning objectives, activities, and assumptions about how the student interacts with the material. According to Bakker and van der Kooij (2020), HLT helps teachers in creating a learning environment that can be tailored to the needs of students, so that learning becomes more responsive and meaningful.

The main problem faced is the existence of discrepancies in the approaches and methodologies used, as well as the limitations of the analysis that maps the patterns of publication and collaboration between researchers. Therefore, this study aims to fill these gaps through bibliometric analysis that can identify global trends, uncover citation and collaboration patterns, and explore themes and gaps in existing literature.

This study aims to provide a deeper insight into the development of research related to *Hypothetical Learning Trajectory*, as well as offering an analysis that is in accordance with the methodology and results of research that has been carried out. With a bibliometric approach, the study will map publication TRENDS, collaboration networks, and the impact of research globally, and also identify gaps that remain unanswered. Such gaps, such as the lack of a thorough understanding of the way mathematical concept abilities are developed and evaluated in various educational contexts, indicate the need for a more comprehensive synthesis.

This research makes a unique contribution by combining two analytical approaches, thus enabling a broader and deeper understanding of the dynamics of research in this field. In addition to providing a clear mapping of trends and collaboration in mathematics learning through the application of *the Hypothetical Learning Trajectory*, the study also provides a foundation for the development of more effective research and educational practices in the future. Thus, this study is not only relevant for researchers, but can also provide guidance for educational policy makers and practitioners in designing learning programs that use this approach.

The purpose of this study was to uncover the evolution of *the Hypothetical Learning Trajectory* in the literature through the analysis of bibliometric data, including the year of publication, the number of citations, and the most frequently used keywords, as

## Hypothetical Learning Trajectory (HLT): Analisis Bibliometrik Berbasis Vos Viewer Bibliometric

well as to identify relevant subjects. It is hoped that the results of this study can make a significant contribution to the scientific literature and advance educational practice in the development of concept abilities in the field of mathematics.

### RESEARCH METHODS

This study applies bibliometric analysis to map the study of the ability of concepts in mathematics learning. Bibliometric analysis is used to explore publication trends and citation patterns derived from scientific journal data indexed in Scopus.

The Data for this study were obtained from the Scopus database using the keyword "*Hypothetical Learning Trajectory*" for the period 2019 to 2024, which was collected on November 20, 2024. This search resulted in 41 articles published with the keyword "*Hypothetical Learning Trajectory*" Scopus is the main source for finding scientific publication data, as it is the largest database in the world that provides accurate information about the metadata of each article, including publication date, abstract, and references (Wan Mohammad & Mohd Azmi, 2023). Data analysis is done using Microsoft Excel, then visualized with VOS Viewer. The use of VOS Viewer aims to present bibliometric maps clearly and easily understood (van Eck & Waltman, 2020). It is hoped that the results of this method can provide a deeper insight *into the Hypothetical Learning Trajectory*, as well as make a significant contribution to the development of research and educational practice in the future.

### RESULTS

The number of articles indexed in Scopus regarding the ability of mathematical concepts from 2019 to 2024 is 41 articles, with the following distribution. In 2019, the number of publications was recorded at 6 articles, and increased to 7 articles in 2020. However, after that, there was a decrease in the number of publications from 2021 to 4 articles, in 2022 and 2023 there were 7 articles and in 2024 it increased to 9 articles.



Figure 1. Publication by year of publication

This Data shows that the trend to examine the theme of krtitis thinking ability has decreased over the past 4 years.

Table 1. Types Of Publications

Year	Conference	article	review
2019	1	5	0
2020	5	2	0
2021	3	1	0
2022	4	2	1
2023	5	2	0
2024	3	6	0

Distribution of publication types indexed by Scopus on the ability of mathematical concepts of publication types, namely articles, conference papers, and reviews from 2019 to 2024.

## Hypothetical Learning Trajectory (HLT): Analisis Bibliometrik Berbasis Vos Viewer Bibliometric

Data shows that in 2019, out of a total of 6 publications, 1 conference paper and 5 articles dominated the publication in that year. Furthermore, in 2020 there was a decrease in the publication of Article 2 and conference papers there was an increase of 5, and there were 3 book chapters. In 2021 the number of articles decreased by 1 and conference papers by 3. In 2022 the decline in articles increased by 28 but conference papers decreased drastically by 3 and *book chapters* also decreased by 1. In 2023 the publication in the form of articles was 2 and conference papers were 1 but there was an increase in, and in 2024 there was a drastic increase in articles by 6 and conference papers by 3.

Publication of articles from the data found to dominate the type of publication in each year, when compared with the publication in the form of conference papers that each year is not fixed. This review shows that the dynamics and focus of publication has changed over the past 5 years, with a view to have increased attention to *the Hypothetical Learning Trajectory*



Figure 2. Quotes every year

The trend of citations for mathematical concept ability in articles indexed in Scopus from 2019 to 2024 shows that in 2019, the number of citations reached 17, which is the highest peak in the last five-year analysis period. In 2020, there was a decrease to 7 quotes. However, publications from previous years experienced significantly decreased recognition, with the number of citations in 2021 dropping to 5. The increase again occurred in 2022 to 14, and a very significant decrease in 2023 to 2024, because no one passed the quotation.

A significant downward trend in citations indicates a change in the focus of research and the number of cited publications. This decline could reflect a change in the relevance of research published in previous years. Therefore, further analysis should be carried out to identify the factors that led to this decline, which may affect future research trends.

Visualizations from the VOS Viewer application will show relationships that are often associated with the ability of mathematical concepts, as well as terms that often appear in research on the ability of mathematical concepts indexed in Scopus from 2019 to 2024.



Figure 3. Network Visualization

## Hypothetical Learning Trajectory (HLT): Analisis Bibliometrik Berbasis Vos Viewer Bibliometric

Based on the network visualization generated from vosviewer analysis, there is a close relationship between several terms in the study of *Hypothetical Learning Trajectory*. Figure 3 shows two main clusters that are distinguished by color: the green cluster that emphasizes the relationship between *the Hypothetical Learning Trajectory* and learning design, and the red cluster that highlights the relationship between concepts and realistic mathematics.

In the green cluster, the relationship between *Hypothetical Learning Trajectory* and mathematics with concepts indicates that *the Hypothetical Learning Trajectory approach* is often used as a tool to improve the ability to understand concepts in the context of mathematical learning. Hal ini konsisten dengan hasil penelitian Yulia (2020) bahwa penerapan *Hypothetical Learning Trajectory* dalam pendidikan matematika dapat meningkatkan kemampuan konsep siswa, dan pentingnya *Hypothetical Learning Trajectory*. The close relationship between these three concepts shows the importance of learning strategies based on *Hypothetical Learning Trajectory* in building concept skills in students.

Meanwhile, the red cluster emphasizes the role of students as the main actors in the development of Concept skills, with particular emphasis on learning. This shows that existing research makes students the center of the learning process that aims to hone their concept skills. A strong relationship between students, concept skills, and problems is very important because Problem-based Learning can improve students' concept skills (Tri & Badraningsih, 2020).



Figure 4. Density Visualization

Based on the visualization of the density map of the VOS viewer shown in Figure 4, there are several terms with high density levels, which indicate their significance and strong frequency in the related literature. The predominance of yellow in some terms, such as concept (Concept), Mathematics (mathematics), *Hypothetical Learning Trajectory* which reflects that these topics are often discussed and become the main focus in research on the ability of mathematical concepts.

This shows that students have a very important role in research on the ability of mathematical concepts published and indexed in Scopus from 2019 to 2024. The ability of mathematical concepts is closely related to students, concepts, and *Hypothetical Learning Trajectory*.

The interrelation between concepts, *Hypothetical Learning trajectories*, and mathematics reinforces the argument that conceptual abilities are often integrated in the context of *Hypothetical Learning trajectories*, especially in mathematics. Results of research by Elbashir, A., Alkhair, Shahad, and Al-Thani, Noora (2024) provide strong evidence that educational programs using *the Hypothetical Learning Trajectory* significantly improve concept skills among high school students. These findings emphasize the importance of incorporating active inquiry-based learning strategies in the learning process that use *the Hypothetical Learning Trajectory* to develop important cognitive skills. The high level of density on these terms suggests that many previous studies have explored the relationship between these aspects.

## Hypothetical Learning Trajectory (HLT): Analisis Bibliometrik Berbasis Vos Viewer Bibliometric

In addition, students and concept ability have a very significant density, which indicates that students are often the main subjects in research that examines concept understanding. This is in line with the literature that places the student as a central actor in the process of developing an understanding of concepts, especially in the context of formal education. In addition, the strong relationship between students, the ability to understand concepts and problems shows that problem solving is still one of the main ways to assess and develop concept skills among students, which is in line with the opinion (Sutama et al. 2022) improvement of concepts based on interpretation indicators is familiarized through problem-based learning.

The study concludes with recommendations for educators and policy makers to develop a more comprehensive framework that supports the integration of concept and problem-solving skills in *the Hypothetical Learning Trajectory of Education*, ultimately preparing students to succeed in the 21st century landscape (Lisnawati, et al, 2024).

This density visualization shows that the ability of mathematical concepts in research is highly dependent on students. thus providing an opportunity for teachers to play a role in improving students ' concept skills through Hypothetical Learning Trajectory learning that is developing today. The findings of this study provide an overview that the ability of the concept has a linkage between students and problems that are in cluster 1 of network visualization analysis, while the concept has a linkage with mathematics, and *Hypothetical Learning Trajectory* is in cluster 2. Paying attention to the ability of mathematical concepts students can use Hypothetical Learning Trajectory learning and by looking at how students solve problems given by the teacher, the picture found can be a reference researchers will further examine some of the focus that has not been seen in the analysis.

Overall, this visualization confirms the importance of concept skills development through the integration of *Hypothetical Learning Trajectory education*. The density of terms seen also shows the direction of research that has been quite well established, but still opens up space for further exploration, especially in relation to innovation of more effective learning methodologies to improve concept skills in students. These visualizations provide valuable insights into research trends and focus, as well as potential gaps for future investigations.

## DISCUSSION

The findings from this bibliometric analysis and review of *the Hypothetical Learning Trajectory* make a significant contribution to the literature that discusses the capabilities of mathematical concepts. From 2019 to 2024, based on data from Scopus, there was an initial increase in publications addressing this topic, but it was followed by a significant decrease in the number of publications and citations since 2021. This decline in publication and citation trends indicates a shift in focus or reduced interest in this topic among researchers, as well as a possible change in the relevance of previous research results to the challenges faced in today's world of Education.

Nonetheless, these findings enrich the understanding of the role of students in concept understanding research, especially related to hypothetical learning design in the analyzed research cluster. Visualization of network and density shows that the relationship between the core learning design of the study. This identification provides new insights into the importance of student engagement in learning mathematical concepts.

However, this study also reveals gaps that need to be explored further, especially regarding the integration of *Hypothetical Learning Trajectory* to improve the ability to understand concepts, as well as new approaches that are more relevant in the context of modern education. Further research is needed to evaluate effective pedagogical interventions in mathematics learning that support improved understanding of concepts with *Hypothetical Learning trajectories* and hypothetical learning designs.

## CONCLUSION

The findings from the bibliometric analysis made a significant contribution to the literature discussing the capabilities of mathematical concepts during 2019 to 2024 with the number of publications 41, the most publications in 2024 as many as 9, and citations as many as 445, but followed by a significant decrease in the number of publications and citations from 2021 to 2024. Some terms that have a high degree of density based on Vos Viewers, show strong significance and frequency in the literature such as *Hypothetical Learning Trajectory*, realistic mathematics,

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## Hypothetical Learning Trajectory (HLT): Analisis Bibliometrik Berbasis Vos Viewer Bibliometric

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