

Teaching Fraction Calculations for Grade 4 Students In Vietnam Oriented Toward the Development of Mathematical Thinking and Reasoning Competency



Ngoc Hoang Bich

Quang Trung Primary Education, Ha Long city, Vietnam.

ABSTRACT: In the context of Vietnam's vigorous general education reform, the current curriculum defines Mathematical Thinking and Reasoning Competency as one of the five components of mathematical competence that students need to achieve. This paper clarifies the manifestations of mathematical thinking and reasoning ability, and proposes a process for teaching fraction calculations to 4th-grade students in a way that fosters this competency. Illustrative examples demonstrate the feasibility and effectiveness of this approach, contributing to the enhancement of math teaching quality and aligning with the current educational reform direction.

KEYWORDS: Mathematical, Calculations with Fractions, Primary School, Mathematics Education.

I. INTRODUCTION

In the current context of educational reform, developing students' competencies—particularly the Mathematical Thinking and Reasoning Competency—is an essential requirement to meet the goals of the 2018 General Education Curriculum. Mathematics instruction should not merely focus on knowledge transmission but must also aim to build and develop students' competencies through active, exploratory, and reasoning-based learning activities. The Organisation for Economic Co-operation and Development (OECD, 2018) emphasizes that Mathematical Thinking and Reasoning Competency is reflected in three key components: (1) identifying, recognizing, organizing, connecting, and representing information; (2) constructing, abstracting, evaluating, and interpreting reasoning; (3) critiquing, rejecting, and revising one's own reasoning [Error! Reference source not found.]. These are core abilities that students must develop in order to apply mathematics effectively in real-life contexts. Gonzalez et al. (2021) define this competency as the ability to conclude, analyze, evaluate, generalize, connect, integrate, and find solutions to problems [Error! Reference source not found.]. Similarly, according to Qomariyah and Darmayanti (2023), it involves the ability to make judgments during problem-solving to reach reasonable conclusions [Error! Reference source not found.]. In Vietnam, the 2018 Mathematics Curriculum also clearly identifies the components of Mathematical Thinking and Reasoning Competency, which include: identifying and articulating mathematical problems in various contexts; analyzing, reasoning, evaluating, and critiquing; modeling and generalizing mathematically. Teaching fractions and related calculations in Grade 4 presents a favorable opportunity to develop these components in students through well-structured, competency-oriented learning activities.

II. CONTENT

A. Mathematical Thinking and Reasoning Abilities

1. Theoretical and Conceptual Framework

- Thinking: According to psychology, thinking is a special property of highly organized matter—the human brain. Thinking reflects the material world through ideal images, revealing internal attributes, the essence, and the logical connections of objects and phenomena previously unknown to the subject. This reflective process is indirect, independent, and abstract in nature, emerging from practical activities and surpassing the limits of sensory perception. According to Pham Minh Hac (1988), "thinking is a psychological process that reflects the essential attributes, connections, and internal laws of objects and phenomena in objective reality that were previously unknown to the subject." [Error! Reference source not found.]. Art Costa (2000) defined thinking as the process of cognition triggered by stimuli from the surrounding environment, received through the senses such as sight, hearing, touch, taste, and smell [Error! Reference source not found.].

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- Reasoning: According to Govier (2010), reasoning is the process by which individuals use arguments and evidence to justify or refute a viewpoint, statement, or hypothesis [**Error! Reference source not found.**]. This process involves constructing premises and drawing conclusions in a logically coherent and rigorous manner.

- This competency plays a vital role in the intellectual development of individuals. It not only helps learners solve mathematical problems but also fosters logical thinking, analytical skills, and problem-solving across various domains. Mathematical Thinking and Reasoning Competency is the ability to use mathematical concepts, methods, and tools to analyze and solve problems. It encompasses identifying, modeling, problem-solving, and critical thinking skills. A student with strong mathematical reasoning skills can approach complex problems by breaking them down into simpler parts and applying mathematical methods to find solutions.

According to the Vietnamese General Education Curriculum, this competency at the primary level includes the following indicators: (i) Performing basic thinking operations, especially the ability to observe, identify similarities and differences in familiar situations, and describe observations; (ii) Providing evidence and logical reasoning before concluding; (iii) Raising and responding to questions during reasoning and problem-solving, initially recognizing evidence and arguments before drawing a conclusion.

- Teaching toward the development of Mathematical Thinking and Reasoning Competency involves organizing learning activities that support students in developing the ability to analyze, generalize, reason logically, critique, and explain within mathematical contexts.

B. Teaching Calculations with Fractions in Direction of Developing Mathematical thinking and reasoning abilities

Instruction designed to cultivate Mathematical Thinking and Reasoning Competency is characterized by the structured orchestration of learning activities aimed at enabling students to develop their abilities in analysis, abstraction, generalization, logical reasoning, critique, and mathematical explanation. Rather than functioning as a conduit for transmitting pre-existing knowledge, mathematics instruction must position learners as active participants in the process of inquiry—encouraging them to identify problems, formulate generalizations, and flexibly apply conceptual understanding to novel contexts. According to the Ministry of Education and Training (2018), Mathematical Thinking and Reasoning Competency is identified as a core disciplinary competency within the mathematics curriculum. It is expressed through students' capacities to observe, analyze, evaluate, generalize, and identify and solve problems arising in both real-life and mathematical situations. This competency is foundational to promoting deep conceptual understanding, fostering logical thought processes, and enhancing the transferability of mathematical knowledge to authentic contexts [**Error! Reference source not found.**]. This perspective is supported by Nguyen Anh (2015), who emphasizes that the development of this competency must be systematically embedded across mathematical domains—especially geometry—to afford students sustained opportunities for engaging in reasoning, justification, and generalization [**Error! Reference source not found.**]. Furthermore, the National Council of Teachers of Mathematics (NCTM, 2000) underscores reasoning and proof as central processes in school mathematics, instrumental for deepening conceptual understanding, connecting mathematical ideas, and cultivating confidence in students' logical thinking [**Error! Reference source not found.**]. Stylianides (2007) posits that to foster students' reasoning, teachers must intentionally design instructional situations that engage learners in mathematical argumentation, discussion, and critique. Such an approach fosters disciplined inquiry, coherent reasoning, and creative problem solving—competencies that are imperative in the 21st-century landscape [**Error! Reference source not found.**].

In alignment with the 2018 Vietnamese General Education Mathematics Curriculum, instructional content in Grade 4 pertaining to fraction calculations encompasses the following topics: addition and subtraction of fractions with like denominators; addition and subtraction of fractions with unlike denominators (where one denominator is a multiple of the other); multiplication and division of fractions; word problems involving fractional operations; and determining a fraction of a given quantity. These curricular components not only consolidate procedural fluency but also provide rich opportunities to develop students' abilities in abstraction, generalization, and mathematical reasoning [**Error! Reference source not found.**]. At this developmental stage, students are transitioning from concrete operational thinking to formal operational thought. As such, instruction on fraction calculations should integrate experiential learning, the use of visual and manipulative aids, and the modeling of real-world contexts. These pedagogical strategies support cognitive engagement and afford students opportunities to analyze, compare, and inductively derive general principles—thereby fostering Mathematical Thinking and Reasoning Competency in a manner that is both meaningful and sustainable. Drawing upon the constituent elements of this competency as articulated in the national curriculum (MOET, 2018), we propose a six-phase instructional framework for teaching fraction operations at the Grade 4 level. This framework is designed to scaffold learners' engagement in problem identification, analytical reasoning, generalization, and the logical formulation of mathematical rules:

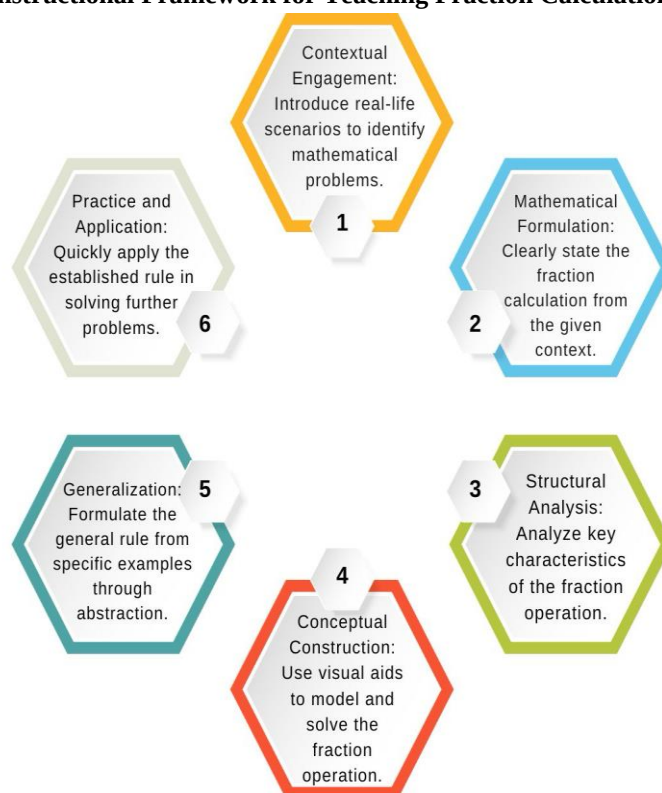
- **Step 1: Contextual Engagement.**

The teacher presents a practical or real-life scenario involving fraction calculations, helping students recognize and articulate the underlying mathematical problem. This initial step is critical in stimulating cognitive curiosity and fostering a problem-solving mindset.

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- **Step 2: Mathematical Formulation**
Students, with guidance from the teacher, clearly define and articulate the mathematical problem emerging from the practical situation. This step involves explicitly identifying the required fraction operation, such as addition, subtraction, multiplication, or division, and clearly stating the mathematical problem to be solved.
- **Step 3: Structural Analysis**
Students observe and analyze the distinctive characteristics and relationships inherent in the fraction operation. Under teacher guidance, students identify critical aspects such as common denominators and relationships between numerators and denominators, thereby deepening their conceptual understanding of the operation.
- **Step 4: Conceptual Construction through Visualization**
Using visual aids, manipulatives, or diagrams, students actively engage in modeling the fraction operation to concretely determine the result. With teacher support, students apply strategies, visualize processes, and accurately interpret the results, bridging concrete experiences with abstract mathematical concepts.
- **Step 5: Generalization and Rule Formation**
Based on their experiences and observations from specific examples, students utilize incomplete induction and abstraction to generalize and formulate mathematical rules or procedures for performing fraction calculations. This final phase consolidates learning, facilitates deep conceptual understanding, and ensures flexible application to new problems
- **Step 6: Practice and Application**
Once the rule is established, the teacher provides students with opportunities to quickly apply the rule to solve several additional fraction calculations.

Table 1: Proposed Five-Step Instructional Framework for Teaching Fraction Calculations



C. Some examples

Example 1: Dạy học phép cộng phân số có cùng mẫu số theo phát triển năng lực tư duy và lập luận toán học

Step 1: The teacher introduces a scenario: Viet, Mai, and Robot are coloring a paper strip. Viet colored $\frac{1}{5}$ and Mai will color another $\frac{2}{5}$. Robot asks, "How many parts have they colored in total?" [Error! Reference source not found.]

Step 2: From this practical scenario, students identify which mathematical operation is required to find the total colored parts, leading to the operation $\frac{1}{5} + \frac{2}{5}$.

Step 3: The teacher writes the mathematical operation on the board and asks students to observe and comment on the terms involved in the addition. From this observation, students recognize that all terms are fractions.

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Then, the teacher asks students to observe and comment on the denominators of the two fractions. Students recognize that both fractions have the same denominator, which is 5. From this observation, the teacher introduces the concept of addition of fractions with same denominators.

Step 4: In this scenario, abstract thinking is demonstrated as students disregard non-essential elements—such as coloring, colors (red and blue), and the paper strip itself—retaining only the essential mathematical elements: $\frac{1}{5}$, $\frac{2}{5}$. Students rely on the keyword "total," which, though commonly used in everyday language, holds significant mathematical meaning in this context as combining. It is from this notion of combining that the concept of addition emerges clearly.

Take a paper strip and divide it into 5 equal parts. The teacher places the paper strip on the board and asks the students: 'When Viet colored $\frac{1}{5}$ of the strip red, how did he do it?' Based on previously learned knowledge, students explain that Viet divided the strip into 5 equal parts and colored one of these parts. The teacher can then demonstrate by coloring one part of the strip red.

"Next, the teacher asks: 'How did Mai color her part?' Students respond that Mai colored two additional parts of the paper strip blue. The teacher invites one student to assist by coloring two more parts on the paper strip displayed on the board. Afterward, the teacher instructs students to observe and determine the total number of parts colored by both students. Students discuss in pairs and conclude that a total of three parts have been colored. The teacher then asks students to explain their reasoning. Students may either count the colored parts directly or logically reason that combining Viet's one part and Mai's two parts results in three parts colored.

From here, the teacher writes on the board and asks students: 'Into how many equal parts is the paper strip divided?' (Answer: 5 parts), 'How many parts have been colored?' (Answer: 3 parts). Then, the teacher asks if any student can write the fraction representing the parts of the strip that have been colored. Students are able to correctly write the fraction $\frac{3}{5}$.

Step 5. The teacher writes the operation $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ on the board. Students observe that the numerator 3 results from adding 1 and 2, while the denominator 5 remains unchanged. The teacher then instructs students to discuss this observation in groups of four. Drawing on their observations, students generalize the rule for adding fractions with like denominators: 'To add fractions with the same denominator, add the numerators and keep the denominator unchanged. [Error! Reference source not found.]

Students then share their ideas. Thus, through abstraction and generalization, students successfully articulate the rule for adding fractions with like denominators.

Step 6: The teacher provides another calculation for students to apply their newly learned knowledge: $\frac{4}{8} + \frac{5}{8}$

Students perform the calculation: $\frac{4}{8} + \frac{5}{8} = \frac{4+5}{8} = \frac{9}{8}$

Example 2: Teaching Multiplication of Fractions Oriented Toward Developing Mathematical Thinking and Reasoning Competency

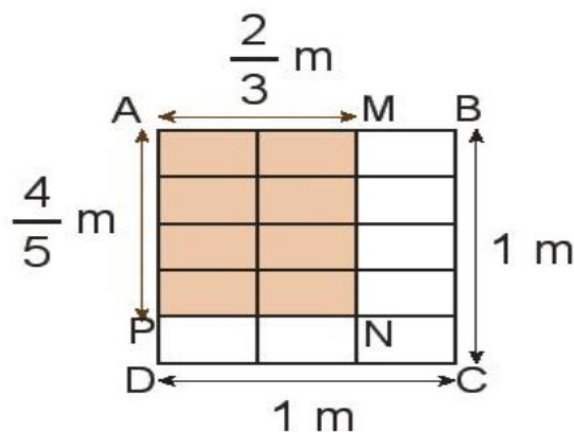
Step 1: The teacher presents an image and organizes students to discuss the following scenario: 'A rectangular glass panel has a length of $\frac{4}{5}m$ and a width of $\frac{2}{3}m$. How can we calculate the area of this rectangular panel? [Error! Reference source not found.]

Step 2: In this situation, students must use abstract thinking and connect to their existing knowledge of calculating the area of a rectangle. Consequently, students articulate that to find the area of the rectangular glass panel, we multiply its length by its width.

Students will then articulate the mathematical operation required to calculate the area of the glass panel: $\frac{4}{5} \times \frac{2}{3}$

Step 3: The teacher organizes students to examine the two factors involved in the above mathematical operation. Students observe and recognize that both factors are fractions. From this observation, students identify and articulate that this is an operation involving the multiplication of two fractions.

Situations: Square ABCD has an area of $1m^2$, divided into 15 equal rectangles, each with an area of $\frac{1}{15}m^2$. Rectangle AMNP covers 8 of these rectangles, having an area equal to $\frac{8}{15}m^2$. Calculate the area of rectangle AMNP?



After establishing the multiplication operation: $\frac{4}{5} \times \frac{2}{3}$ the teacher organizes an activity for the students.

Step 4: First, the teacher poses the question: “How do we obtain $\frac{4}{5}m$ ” Students respond that a 1-meter length must be divided into 5 equal parts, with 4 parts taken to represent $\frac{4}{5}m$. Similarly, to obtain $\frac{2}{3}m$ the 1-meter length should be divided into 3 equal parts, and 2 parts are taken. Now, to construct a rectangle with a length of $\frac{4}{5}m$ and a width of $\frac{2}{3}m$ we will start with a square.

The teacher presents the following task: “I have a square ABCD with sides measuring 1 meter. Could a student help me obtain $\frac{4}{5}m$ along side AD?”. Students explain that side AD should be divided into 5 equal parts, and 4 of these parts will represent $\frac{4}{5}m$. The teacher marks this point as P. Similarly, to obtain $\frac{2}{3}m$ along side AB, students suggest dividing side AB into 3 equal parts and taking 2 parts to identify point M. After marking these points, the teacher invites a student to the board to color the rectangle formed with a length of $\frac{4}{5}m$ and a width of $\frac{2}{3}m$.

Next, the teacher asks students to calculate the area of square ABCD and to determine into how many smaller rectangles the square has been divided and why. Students count or calculate and identify 15 smaller rectangles. The teacher then asks for the area of each smaller rectangle. Since the square with an area of $1m^2$ is divided into 15 rectangles, each rectangle has an area of $\frac{1}{15}m$. Rectangle AMNP covers 8 of these rectangles.

Step 5:

The teacher analyzes the numbers $15 = 5 \times 3$; $8 = 4 \times 2$. Thus, rectangle AMNP includes 8 smaller rectangles, each with an area of $\frac{1}{15}m^2$. The teacher asks students to state the area of rectangle AMNP. Students respond with $\frac{8}{15}m^2$. The teacher then reverses the question to students: “What is the result of $\frac{4}{5} \times \frac{2}{3}$?” Students correctly state the result as $\frac{8}{15}$. The teacher writes this operation on the board and instructs students to discuss in groups of four to generalize and formulate the rule for multiplying two fractions.

Step 6: The teacher provides another calculation for students to apply their knowledge: $\frac{4}{8} + \frac{5}{8}$

Students perform the calculation: $\frac{4}{8} + \frac{5}{8} = \frac{4+5}{8} = \frac{9}{8}$

Through these activities, the development of mathematical thinking and reasoning competency is evident as students apply existing knowledge to articulate the operation for calculating the area of a rectangle, whose length and width measurements are fractions rather than whole numbers, as previously learned. Through abstraction and generalization, students effectively formulate the general rule for multiplying fractions: “To multiply fractions, multiply the numerators and multiply the denominators.”

III. CONCLUSIONS

In the current educational context emphasizing competency-based reform, the teaching of mathematics—and specifically fraction calculations—must prioritize the cultivation of students' mathematical thinking, reasoning competencies, and problem-solving abilities. Rather than merely disseminating knowledge, educators should design and implement exploratory, experiential, and analytical learning activities that engage students in meaningful inquiry. This approach enables students to deeply comprehend mathematical principles, thus fostering logical reasoning and adaptive application skills. Grounded in the theoretical framework provided by the 2018 Mathematics Curriculum and relevant educational research, this paper proposes a structured six-step instructional model, beginning with authentic situational contexts and culminating in the generalization of mathematical rules via incomplete induction. Illustrative examples validate the effectiveness of this instructional approach in developing mathematical thinking and reasoning competencies among Grade 4 students. Consequently, learners are empowered not only to master

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computational procedures but also to cultivate essential skills such as analytical thinking, explanatory reasoning, generalization, and adaptive problem-solving. The outcomes of this research affirm that competency-oriented teaching of fraction calculations aligns significantly with current educational reform objectives and contributes substantially to enhancing the overall quality of primary mathematics education, particularly at the Grade 4 level. Therefore, this instructional strategy merits broad endorsement and widespread adoption in educational practice.

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