

Designing and Using Number and Arithmetic Exercises in Grade 4 to Contribute to the Development of Students' Mathematical Communication Competence

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ABSTRACT: Numbers and arithmetic constitute a core content strand in the primary school Mathematics curriculum. In Grade 4, the strand Numbers and Arithmetic includes knowledge of natural numbers and operations with natural numbers, fractions, and operations with fractions. This paper presents research findings and proposes a process for designing and using number and arithmetic exercises in Grade 4 to contribute to the development of students' mathematical communication competence. In addition, the paper illustrates four examples applying the proposed process to the design and implementation of exercises. Each example corresponds to a different topic within the Numbers and Arithmetic content in Grade 4.

KEYWORDS: communication, mathematical communication, numbers and arithmetic, Grade 4

1. INTRODUCTION

Competency-based teaching and learning has been widely implemented in Vietnamese schools. Mathematical communication competence is one of the five component competencies of mathematical competence specified in the General Education Curriculum for Mathematics. In mathematics teaching, teachers are expected not only to help students acquire mathematical knowledge and skills but also to emphasize the development of competencies in general and mathematical communication competence in particular.

This raises an important question: How should teachers design and use number and arithmetic exercises to effectively contribute to the development of students' mathematical communication competence? To address this question, the authors conducted a study and present a summary of the research findings in this paper.

2. MATHEMATICAL COMMUNICATION COMPETENCE

2.1. Some Basic Concepts

- Exercises are instructional tools used in the teaching and learning process, through which teachers require students to perform specific tasks in order to achieve particular learning objectives. Exercises help students better understand mathematical knowledge and apply that knowledge to problem solving. They also serve as a means to promote self-directed learning and time management in learning activities.

Different types of exercises serve different instructional purposes and learning contents. Common types of exercises at the primary level include practice exercises, application exercises, and situational or contextualized exercises.

- Competence is defined as an individual attribute formed and developed through innate qualities and through learning and training processes, enabling a person to mobilize and integrate knowledge, skills, and other personal attributes such as interest, belief, and willpower to successfully perform a certain type of activity and achieve desired outcomes under specific conditions [1].

- Communication is understood as the process of interaction between individuals through spoken language, written language, and non-verbal expressions to exchange information, interact, and engage in discussion. Communication involves exploring and understanding the specific characteristics of the information exchange process between participants.

- Communication competence can be understood as an individual's ability to convey and receive information, emotions, and ideas effectively in various situations. It includes the ability to use spoken language to express ideas clearly, coherently, persuasively, and engagingly; the ability to listen, comprehend information, and respond appropriately; the ability to express ideas accurately and clearly in written language; and the ability to demonstrate confidence and flexibility in communication.

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In primary mathematics education, mathematical communication competence plays an essential role in helping students deepen their understanding of mathematical concepts, develop logical thinking, and form positive learning attitudes toward mathematics.

2.2 Mathematical Communication Competence

2.2.1. Conceptual framework

Mathematical communication is the process of information exchange in mathematics lessons between the teacher and individual students, among individual students, between individual students and the whole class, and between the teacher and the class as a whole, with the aim of achieving mathematics learning objectives. The tools used in mathematical communication include mathematical language and natural language.

Mathematical competence is defined as “an individual’s ability to formulate, apply, and interpret mathematics in a variety of contexts”; it “includes mathematical reasoning and the use of concepts, procedures, facts, and tools to describe, explain, and predict phenomena; helps individuals recognize the role of mathematics in the world and make well-founded judgments and decisions as constructive, engaged, and reflective citizens.”

Mathematical communication competence can be understood as students’ ability to use mathematical language to read and write mathematical texts and to exchange mathematical information during mathematics lessons.

2.2.2. Manifestations of mathematical communication competence at the primary level [2]

Competence components	Manifestations at the primary level
Mathematical communication competence is demonstrated through the ability to:	
“Listen to, read, understand, and record necessary mathematical information presented in mathematical texts or conveyed orally or in writing by others.”	“Listen to, read, understand, and record (summarize) key mathematical information in texts or information communicated by others (at a simple level), thereby identifying the problem to be solved.”
“Present and express (orally or in writing) mathematical contents, ideas, and solutions through interaction with others (with appropriate requirements for completeness and accuracy).”	“Present and express (orally or in writing) mathematical contents, ideas, and solutions through interaction with others (without requiring full completeness or accuracy); pose and respond to questions during reasoning and problem solving.”
“Use mathematical language effectively in combination with everyday language or gestures when presenting, explaining, and evaluating mathematical ideas through interaction with others.”	“Use mathematical language combined with everyday language and gestures to express mathematical content in simple situations.”
“Demonstrate confidence when presenting, expressing ideas, asking questions, discussing, and debating mathematical content.”	“Demonstrate confidence when answering questions and when presenting and discussing mathematical content in simple situations.”

3. DESIGNING AND USING NUMBER AND ARITHMETIC EXERCISES TO DEVELOP STUDENTS' MATHEMATICAL COMMUNICATION COMPETENCE

3.1. Principles for designing and using exercises

Principle 1. Exercises must ensure alignment with content and expected learning outcomes Designed exercises must meet the expected learning outcomes of Grade 4 Mathematics in general, and of the Numbers and Arithmetic content strand in particular. Exercises should aim not only to reinforce students’ mathematical knowledge and skills but also to create opportunities for competence development.

Principle 2. Exercises must ensure practicality and suitability for primary school students Exercises should be connected to real-life situations that students may encounter in their daily lives. This helps students recognize opportunities to apply mathematical knowledge to practical problem solving.

Exercises must be appropriate for Grade 4 students. They may provide or extend students’ understanding of economic, cultural, and social contexts of the country, but should not be unfamiliar or overly abstract.

Principle 3. Exercises must provide opportunities to develop mathematical problem-solving competence Exercises should require students to analyze, compare, and justify their choices. Students are given opportunities to read and comprehend mathematical content and to identify and apply the necessary mathematical knowledge when solving problems.

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Principle 4. The use of exercises should foster a collaborative and creative learning environment. Students are encouraged to work in groups to solve problems, thereby developing communication and collaboration skills. Multiple solution strategies for the same problem are encouraged, fostering creative thinking.

Students should be given opportunities to present their own solutions, as well as to comment on, evaluate, and critique solutions proposed by their peers.

3.2. Process of designing and using exercises

Step 1. Identifying the content and expected learning outcomes of the topic. Based on the content and expected learning outcomes specified in the 2018 Mathematics Curriculum for Grade 4, detailed content is identified according to the learning objectives of the topic. From this basis, suitable content can be selected for designing exercises that incorporate practical elements and are oriented toward real-life contexts.

Step 2. Designing exercises oriented toward developing students' mathematical communication competence. Appropriate real-life contexts for Grade 4 students are identified. The contexts should be familiar to students and may provide additional information related to local or national economic, cultural, or social conditions, while remaining accessible and meaningful.

Exercise design should emphasize the application of mathematical knowledge to solving problems with practical (realistic or hypothetical) elements.

During the design process, it is necessary to analyze how each exercise creates opportunities for students to develop mathematical communication competence.

Step 3. Using the designed exercises in teaching to develop students' mathematical communication competence. Appropriate forms of organizing learning activities using the designed exercises are proposed in alignment with the goal of developing mathematical communication competence. The necessary mathematical knowledge required to solve each problem is identified.

Students are guided to read the problem, exchange ideas, and discuss with peers to explore solution strategies; to express their opinions orally or in writing; and to use mathematical language to present their solutions accurately and logically.

3.3. Illustrative Examples

Example 1. Designing and Using Exercises on "Rounding Natural Numbers to the Hundred-Thousands Place"

Step 1. Identifying the content and expected learning outcomes of the topic

Based on the expected learning outcome "Students are able to round numbers to the nearest ten, hundred, thousand, ten thousand, and hundred thousand" [2], the specific content is identified as rounding natural numbers to the hundred-thousands place. The mathematical content that can be applied to solving real-life problems is rounding natural numbers.

Step 2. Designing exercises to contribute to the development of students' mathematical communication competence

Round the population of each of the following provinces [3] to the nearest hundred thousand and fill in the blanks:



Dien Bien: 673,091 people



Son La: 1,404,587 people



Lao Cai: 1,778,785 people



Lai Chau: 512,061 people

The population of Dien Bien Province is approximately people.

The population of Son La Province is approximately people.

The population of Lao Cai Province is approximately people.

The population of Lai Chau Province is approximately people.

Analysis of opportunities for developing mathematical communication competence: This exercise provides students with population data of several provinces at a specific point in time. It incorporates authentic real-life elements, allowing students to apply knowledge of rounding numbers to solve a practical problem. At the same time, the exercise offers students additional social

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knowledge. By using rounding techniques, students are able to express approximate population figures of a province without having to remember the exact population number. Thus, reading and solving the problem not only helps students practice reading comprehension and selecting relevant mathematical information, but also enhances their understanding of population characteristics of the localities mentioned in the exercise.

Step 3. Using the designed exercise in teaching to develop students' mathematical communication competence

Students read the problem and identify the task to be solved: rounding natural numbers to the hundred-thousands place, then use the rounding results to describe the approximate population of each province given in the problem.

Students discuss and exchange ideas with peers to determine how to round numbers to the hundred-thousands place: identifying the hundred-thousands digit, identifying the ten-thousands digit, and then applying the rounding rule for natural numbers to obtain the result.

Individual activity: Students complete the exercise.

The population of Dien Bien Province is approximately 700,000 people;

The population of Son La Province is approximately 1,400,000 people;

The population of Lao Cai Province is approximately 1,800,000 people;

The population of Lai Chau Province is approximately 500,000 people.

Pair work: Students share their answers with a partner, reach agreement on the results, and engage in question-answer exchanges about the solution process.

Student A: Explain why the population of Điện Biên Province is approximately 700,000 people.

Student B: Rounding 673,091 to the nearest hundred thousand is done as follows: the digit in the hundred-thousands place is 6; the digit in the ten-thousands place is 7. Comparing 7 with 5, we have $7 > 5$. Therefore, when rounding 673,091 to the nearest hundred thousand, we round up to obtain 700,000. Hence, the population of Dien Bien Province is approximately 700,000 people.

Whole-class activity: A student representative presents the solution and invites classmates to comment and ask questions. The teacher provides feedback and confirms the correct results of the exercise.

Example 2. Designing Exercises on Reading and Writing Fractions

Step 1. Identifying the content and expected learning outcomes of the topic

Based on the expected learning outcomes, we identify the specific content for designing fraction-related exercises, including reading and writing fractions. Students observe visual representations in which a whole is divided into equal parts, some parts are taken away, or a certain number of parts remain.

Exercises are designed using images closely related to real-life situations so that students can see how mathematics is applied to solving everyday problems. At the same time, when students read the problem statements and use mathematical language in combination with natural language to exchange ideas and discuss with peers, their mathematical communication competence is fostered.

Step 2. Designing exercises to contribute to the development of students' mathematical communication competence

Write the fraction representing the remaining part of the cake in each of the following figures:



Analysis of opportunities for developing mathematical communication competence: The problem presents images commonly encountered in daily life, such as sharing cakes. Students observe the visual representations and identify the task to be solved: each cake is divided into equal parts, and some parts have been eaten or removed. Students determine the number of remaining parts and write the corresponding fraction.

This exercise contributes to the development of mathematical communication competence by enabling students to read mathematical information presented visually and to write fractions representing the remaining parts of each cake.

Step 3. Using the designed exercise in teaching to develop students' mathematical communication competence

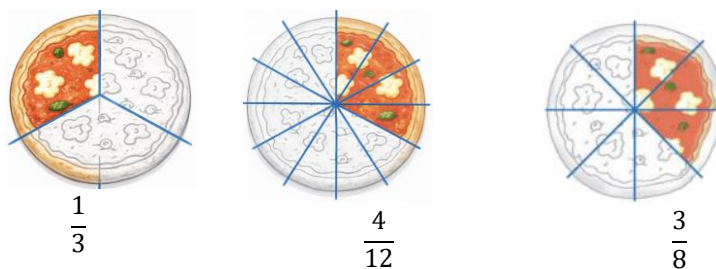
Students work individually to observe the images and read the mathematical information conveyed in the figures. They identify the task to be solved: writing fractions to represent the remaining part of each cake.

Students discuss and carefully observe each cake to determine how many equal parts each cake is divided into and how many parts remain.

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Students explain their solution process. For example, observing that the first cake is divided into three equal parts, with one part remaining, leads to the conclusion that $\frac{1}{3}$ of the cake remains.

Students present their results:



Students use mathematical language combined with natural language to explain their solutions to peers. For example: There is one cake divided into three equal parts; after eating some, one part remains. The fraction representing the remaining part of the cake is $\frac{1}{3}$.

Students may ask follow-up questions, such as: How many parts of the cake have been eaten?

Students observe the image, think, and explain: There is one cake divided into three equal parts; two parts have been eaten. The fraction representing the part of the cake that has been eaten is $\frac{2}{3}$.

Example 3. Designing Practice Exercises for Problem Solving Involving Multiplication of Natural Numbers by Two-Digit Numbers

Step 1. Identifying the content and expected learning outcomes of the topic

Based on the expected learning outcomes, we identify the specific content for designing exercises on fractions, including: solving word problems related to the multiplication of multi-digit natural numbers by two-digit numbers; and understanding the relationship between the units “yến”¹ and kilogram.

Step 2. Designing exercises to contribute to the development of students' mathematical communication competence

Did you know?

Dien Bien Tam Rice is well known for its attractive aroma, delicious taste, and soft, sticky texture. Điện Biên Tam Rice is grown in the fields of the Mường Thanh valley using traditional manual methods from planting and cultivation to harvesting.

Designing exercises

Ms. Loan buys 4 yen of Dien Bien Tam Rice as gifts. The price of 1 kilogram of Dien Bien Tam Rice is 22,000 VND. How much does Ms. Loan have to pay for this amount of rice?

Analysis of opportunities for developing mathematical communication competence: The problem provides students with information about a local specialty – Dien Bien Tam Rice. It presents a hypothetical situation that allows students to apply mathematical knowledge to problem solving. By reading the problem, students understand the mathematical information conveyed. They identify the given information: “Ms. Loan buys 4 yen of Dien Bien Tam Rice. The price of 1 kilogram of rice is 22,000 VND,” and the information to be found: “How much does Ms. Loan have to pay for this amount of rice?”. Students recognize the mathematical problem to be solved: calculating the total cost of purchasing 4 yen of rice.

Step 3. Using the designed exercise in teaching to develop students' mathematical communication competence

Students are organized to read the problem and underline the given information and the information to be found.

Ms. Loan buys 4 yen of Dien Bien Tam Rice as gifts. The price of 1 kilogram of rice is 22,000 VND. How much does Ms. Loan have to pay for this amount of rice?

Students summarize the problem:

1 kg: 22,000 VND

4 yen: ... VND?

Students discuss and propose solutions to the problem.

They realize that the first step is to convert 4 yen into kilograms. Then, they perform a multiplication to find the total cost by multiplying the price per kilogram by the number of kilograms purchased.

Students present their solution:



Highland rice fields

¹ “yến” means 10 kilogram in Vietnamese.

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Solution

Convert 4 yen of rice into kilograms: 4 yen = 40 kg

The amount of money Ms. Loan has to pay is:

$$22,000 \times 40 = 880,000 \text{ (VND)}$$

Answer: 880,000 VND.

Example 4. Designing Exercises Related to Finding a Fraction of a Number

Step 1. Identifying the content and expected learning outcomes of the topic

Based on the expected learning outcome “Students are able to solve problems related to the four operations with fractions,” we identify the specific content as solving problems by finding a fraction of a number.

Step 2. Designing exercises to contribute to the development of students' mathematical communication competence:

Mr. Tu buys a roll of electrical wire as shown in the figure. On the first day, he uses $\frac{1}{6}$ of the roll; on the second day, he uses $\frac{2}{3}$ of the roll. How many meters of electrical wire does Mr. Tu use in total over the two days?



Analysis of opportunities for developing mathematical communication competence:

The problem presents an authentic real-life image—a roll of electrical wire with a length of 60 meters—combined with a hypothetical context. Students apply mathematical knowledge related to finding a fraction of a number and adding fractions to solve the problem. The context is familiar and close to students' daily experiences, creating opportunities for students to develop mathematical problem-solving competence.

Step 3. Using the designed exercise in teaching to develop students' mathematical communication competence

When students are first introduced to solving problems involving finding a fraction of a number, the problem can be broken down into simpler sub-problems to make it easier to understand. For example:

- Find $\frac{1}{6}$ of 60 meters.
- Find $\frac{2}{3}$ of 60 meters.
- Find the total number of meters of wire used over the two days.

Alternatively, the problem can be reformulated as:

- “On the first day, Mr. Tu uses $\frac{1}{6}$ of the roll; on the second day, he uses $\frac{2}{3}$ of the roll. What fraction of the roll is used over the two days?”

- “A roll of electrical wire is 60 meters long. Mr. Tu has used $\frac{5}{6}$ of the roll. How many meters of wire has he used?”

Students read the problem and underline the key information: “Mr. Tu buys a roll of electrical wire. On the first day, he uses $\frac{1}{6}$ of the roll; on the second day, he uses $\frac{2}{3}$ of the roll. How many meters of electrical wire does Mr. Tu use in total over the two days?”

Students identify the problem to be solved: finding the total number of meters of wire used over the two days.

Students engage in question – answer exchanges about the information provided in the problem:

- What information is given? (Mr. Tu buys a roll of electrical wire. On the first day, he uses $\frac{1}{6}$ of the roll; on the second day, he uses $\frac{2}{3}$ of the roll.)

- What is the problem asking? (How many meters of electrical wire does Mr. Tu use in total over the two days?)

Students discuss and find a solution. One possible solution is presented as follows:

Solution

The fraction of the roll used over the two days is:

$$\frac{1}{6} + \frac{2}{3} = \frac{5}{6} \text{ (of the roll)}$$

The number of meters of wire used is:

$$\frac{5}{6} \times 60 = 50 \text{ (m)}$$

Answer: 50 meters of electrical wire.

Organize a discussion among students to find alternative solutions to the problem. Students could solve this problem as follows:

On the first day, the amount of wire used is:

$$60 \times \frac{1}{6} = 10 \text{ (m)}$$

On the second day, the amount of wire used is:

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$$60 \times \frac{2}{3} = 40 \text{ (m)}$$

The total amount of wire used over the two days is:

$$10 + 40 = 50 \text{ (m)}$$

Answer: 50 meters of electrical wire.

The students were asked to compare the two solutions and indicate which one was faster.

4. CONCLUSION

In mathematics teaching, teachers need to design exercises that not only help students consolidate mathematical knowledge and skills but also provide additional understanding of fields related to real-life practice. Well-designed problems should create opportunities for students to develop mathematical communication competence through reading and understanding mathematical content, as well as presenting and expressing ideas in spoken or written language. Students are given opportunities to comprehend the information presented in the problems, exchange ideas with peers to find solutions, and present their own methods to others. They use mathematical language and natural language during discussion, debate, or when presenting solutions. The procedure proposed in this article has been applied and implemented by primary school teachers in the design and use of exercises within the knowledge strand Numbers and Operations in Grade 4 in an effective manner.

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