

Some Pedagogical Measures for Teaching the Topic “Vector Coordinates in Space” Toward Developing Mathematical Modeling Competency for Grade 12 Students

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ABSTRACT: Mathematical modeling competency is one of the essential specific competencies that need to be developed for upper secondary school students in accordance with the orientation of the 2018 General Education Curriculum in Vietnam. In Grade 12 mathematics teaching, the topic *Vector Coordinates in Space* offers considerable potential for organizing learning activities connected to real-life contexts, thereby fostering students' mathematical modeling competency. However, classroom practice indicates that exploiting this topic from a competency-based perspective remains limited, particularly in the design and implementation of modeling situations. This paper presents research results on proposing several pedagogical measures for teaching the topic *Vector Coordinates in Space* with the aim of developing mathematical modeling competency for Grade 12 students. Based on an analysis of theoretical perspectives on competency, mathematical modeling competency, and the expected learning outcomes of the topic in the current mathematics curriculum, the study develops teaching measures that support students in performing the stages of the mathematical modeling process. These measures are designed to enhance learning activities associated with real-world contexts and to promote students' active and autonomous engagement in learning. The research findings contribute both theoretical and practical foundations for mathematics teachers in organizing competency-oriented teaching to develop students' mathematical modeling competency in upper secondary schools.

KEYWORDS: mathematical modeling competency; vector coordinates in space; mathematics teaching; Grade 12 students.

I. INTRODUCTION

In the context of comprehensive educational reform oriented toward developing learners' qualities and competencies, mathematics teaching in general education aims not only to equip students with knowledge and skills but also to foster the development of core mathematical competencies. The 2018 General Education Curriculum of Vietnam identifies mathematical modeling competency as one of the five specific competencies of mathematics, playing a crucial role in helping students identify, analyze, and solve problems arising from real-life contexts through mathematical tools and methods (Ministry of Education and Training [MOET], 2018).

Numerous international studies have affirmed that mathematical modeling is not merely a means of connecting mathematics with the real world but also an effective pathway for developing higher-order thinking and problem-solving competencies. According to Blum and Leib (2006), mathematical modeling enables students to understand the role of mathematics in interpreting, predicting, and making decisions in various contexts of life and science. Niss (2003) emphasized that mathematical modeling competency is manifested in the ability to flexibly translate between real-world situations and mathematical models, as well as to evaluate the reasonableness of obtained results. The OECD's PISA 2018 report further highlighted that students' mathematical competency is closely associated with their capacity to formulate, apply, and interpret mathematics in diverse real-world situations (OECD, 2019). In Vietnam, mathematical modeling in mathematics education has attracted considerable attention from researchers. Nguyễn Thị Tân An (2012) pointed out that incorporating mathematical modeling into mathematics teaching is essential for helping students grasp the nature of mathematical knowledge and enhancing their ability to apply mathematics in practice. Nguyễn Thị Nga (2014) argued that modeling-oriented teaching contributes to pedagogical innovation by promoting students' activeness and autonomy in learning. Nguyễn Danh Nam (2015, 2016) clarified the modeling process and emphasized the role of mathematical modeling in developing students' mathematical competencies. More recent studies have focused on proposing pedagogical measures to foster mathematical modeling competency through specific topics in the upper secondary mathematics curriculum (Nguyễn Thị Nga, 2023; Cao Thị Hà & Nguyễn Xuân Dung, 2023).

Within the upper secondary mathematics curriculum, the topic *Vector Coordinates in Space* is an important component of Grade 12 mathematics, with wide applications in physics, engineering, and real-life contexts. This topic provides effective mathematical tools

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for describing positions, directions, and geometric relationships in space, while also offering favorable conditions for organizing learning activities oriented toward mathematical modeling. However, classroom practice indicates that exploiting this topic to develop students’ mathematical modeling competency remains limited. Most teaching activities still focus on knowledge transmission and computational skill training, whereas real-life situations with high modeling potential have not been systematically designed and implemented.

Arising from the demand for renewing mathematics teaching toward a competency-based approach and from difficulties observed in teaching practice, this paper focuses on proposing several pedagogical measures for teaching the topic *Vector Coordinates in Space* toward developing mathematical modeling competency for Grade 12 students. These measures are developed based on theoretical foundations of mathematical modeling competency and the content requirements of the current mathematics curriculum, with the aim of supporting teachers in organizing effective instruction and contributing to improving the quality of mathematics teaching in upper secondary schools.

II. RESEARCH METHODS

This study primarily employed theoretical and practical research methods. Specifically, document analysis, synthesis, and systematization were used to clarify key concepts related to competency and mathematical modeling competency, as well as to establish the theoretical foundation for proposing pedagogical measures oriented toward competency development. In addition, the 2018 General Education Mathematics Curriculum and the content of the topic *Vector Coordinates in Space* in Grade 12 were analyzed to identify expected learning outcomes and the potential for developing students’ mathematical modeling competency. Furthermore, practical research methods such as classroom observation, questionnaire surveys, and expert consultation were conducted to investigate the current state of teaching this topic in upper secondary schools, thereby providing a basis for proposing feasible and appropriate pedagogical measures.

III. RESEARCH RESULTS

3. Research Results

3.1. Mathematical Modeling Competency

3.1.1. Concepts

In contemporary educational approaches, competency is understood as the ability to mobilize and integrate knowledge, skills, attitudes, and other personal attributes to effectively perform a particular type of activity in specific contexts. According to the 2018 General Education Curriculum, competency is reflected not only in learning outcomes but also in the process by which learners apply their knowledge to solve problems arising in learning and everyday life (MOET, 2018).

Based on the concept of competency, numerous scholars have approached and clarified the notion of mathematical modeling competency. Blum and Leib (2006) defined mathematical modeling competency as the ability to move back and forth between real-world situations and mathematical models, including simplifying situations, constructing models, solving models, and interpreting results within real-life contexts. Niss (2003) emphasized that mathematical modeling competency extends beyond model construction to include the ability to evaluate, adjust, and validate the appropriateness of the constructed models.

In Vietnam, Nguyễn Thị Nga (2014) described mathematical modeling competency as students’ ability to use mathematical tools to describe, analyze, and solve problems related to real-life contexts. Nguyễn Danh Nam (2015, 2016) viewed mathematical modeling competency as a multi-stage process in which students actively engage in understanding real-world problems, constructing mathematical models, solving these models, and interpreting the results.

In this study, mathematical modeling competency is understood as the ability of upper secondary school students to identify real-life problems that can be addressed mathematically, construct and use appropriate mathematical models, solve these models, and evaluate the results within real-world contexts, in accordance with the orientation of the 2018 General Education Mathematics Curriculum.

3.1.2. Manifestations of Mathematical Modeling Competency of High School Students

Based on both international and domestic studies (Blum & Leib, 2006; Niss, 2003; Nguyễn Danh Nam, 2015), mathematical modeling competency of upper secondary school students can be manifested through the following core components:

- **Identifying and clarifying real-world problems:** Students are able to determine relevant elements, quantities, and relationships in real-life situations and recognize problems that can be addressed using mathematical tools and methods.
- **Constructing mathematical models:** Students select and build appropriate mathematical models (such as expressions, equations, vectors, or coordinate systems) to represent real-world situations.
- **Solving mathematical models:** Students apply mathematical knowledge and skills to process the constructed models, performing necessary transformations, calculations, and logical reasoning.

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- **Interpreting and evaluating results:** Students interpret mathematical results in the original real-life context, assess the reasonableness of the results, and adjust the models when necessary.

These manifestations are closely interconnected and carried out in a continuous cycle. Organizing teaching toward developing mathematical modeling competency should provide students with opportunities to experience all stages of this process through learning situations connected to real-life contexts, especially in topics with high application potential such as *Vector Coordinates in Space* in Grade 12.

3.1.3. Stages of the Mathematical Modeling Process in Teaching *Vector Coordinates in Space*

Stage 1. Identifying the problem and transforming the situation

Students are able to analyze spatial geometry situations (such as angles, distances, and relative positions of lines and planes) and recognize that they can be expressed mathematically through coordinate systems and vectors. Students learn to eliminate secondary factors while retaining essential data to construct spatial geometric models.

Stage 2. Constructing the mathematical model

Students select appropriate coordinate systems and represent points, vectors, lines, and planes using coordinates. Based on this representation, they establish equations, systems of equations, or vector expressions to describe geometric relationships such as parallelism, perpendicularity, and coplanarity.

Stage 3. Solving the model

Students apply computational tools (dot product, cross product, parametric and canonical equations) to obtain solutions. At the same time, they integrate algebraic and geometric knowledge to address complex situations, such as calculating volumes of polyhedra or distances between skew lines.

Stage 4. Interpreting and evaluating results and proposing optimal models

Students interpret mathematical solutions as geometric or real-life conclusions (e.g., determining lengths, areas, angles, or volumes). They verify the reasonableness of results by comparing them with diagrams or real conditions. From specific problems, students generalize solution methods for similar situations and relate vector-coordinate models to real-world applications or other subjects such as Physics and Informatics.

3.2. The Topic “*Vector Coordinates in Space*” (Grade 12)

The topic *Vector Coordinates in Space* is an important component of the Grade 12 mathematics curriculum, belonging to the strand of analytic geometry in space. This topic provides students with essential mathematical tools to describe positions, directions, and geometric relationships among objects in three-dimensional space through coordinate systems and vectors. It also serves as a foundation for students to approach and solve various applied problems in fields such as physics, engineering, and real-life contexts. According to the 2018 General Education Mathematics Curriculum, through studying the topic *Vector Coordinates in Space*, students are expected to achieve key knowledge and skills, including determining vector coordinates in a three-dimensional coordinate system; performing vector operations using coordinates; and applying the dot product of vectors to solve problems involving angles, distances, and geometric relationships in space. In addition, the curriculum emphasizes the organization of learning activities aimed at developing mathematical competencies, particularly mathematical modeling competency (MOET, 2018).

In terms of content, this topic comprises core knowledge such as the three-dimensional coordinate system, vector coordinates, vector operations in space, the dot product of vectors, and their applications. These concepts are closely interconnected and allow for flexible transitions between geometric representations and algebraic representations, thereby creating favorable conditions for constructing and exploiting mathematical models.

From the perspective of developing mathematical modeling competency, the topic *Vector Coordinates in Space* offers significant potential for organizing learning activities connected to real-life contexts. Through situations involving the determination of positions, directions of motion, distances, or angles between objects in space, students have opportunities to experience all stages of the mathematical modeling process—from identifying real-world problems, constructing mathematical models using vectors and coordinates, solving the models, to interpreting and evaluating the results. However, to effectively exploit this potential, teachers need appropriate pedagogical measures to guide and support students throughout the learning process.

3.3. Pedagogical Measures for Teaching the Topic “*Vector Coordinates in Space*” toward Developing Mathematical Modeling Competency for Grade 12 Students.

3.3.1. Measure 1. Organizing Teaching through Real-Life Situations to Help Students Identify Problems and Construct Mathematical Models

Purpose:

This measure aims to develop students’ ability to recognize real-life situations that can be addressed mathematically, identify relevant quantities and relationships, and initially construct appropriate mathematical models within the topic *Vector Coordinates in Space*.

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Implementation:

Step 1. Selecting and designing real-life situations.

The teacher selects familiar contexts involving spatial elements (such as position, direction, or displacement) that provide sufficient data for modeling using vectors and coordinates. These situations should be closely related to students' experiences and suitable for representation in three-dimensional space.

Step 2. Guiding students to analyze the situation.

Students observe or read the given situation, determine the problem to be solved, and list the objects, quantities, and relationships involved. This step helps students focus on essential information while understanding the context of the problem.

Step 3. Standardizing language and formulating assumptions.

Through guiding questions, the teacher supports students in establishing reasonable assumptions (e.g., neglecting secondary factors, choosing appropriate units, defining positive directions) and expressing the situation using mathematical language.

Step 4. Selecting tools and constructing the mathematical model.

Students choose a suitable coordinate system, identify relevant points and vectors, represent them using coordinates, and transform the real-life situation into a mathematical problem based on vector-coordinate representations.

Step 5. Verifying the appropriateness of the model.

Before solving the model, students compare the constructed mathematical model with the original situation to check whether all essential data have been represented correctly and whether the model is appropriate for addressing the problem.

3.3.2. Measure 2. Guiding Students to Solve and Exploit Mathematical Models through Active Learning Activities

Purpose:

This measure aims to develop students' ability to solve mathematical models by applying knowledge and skills related to vector coordinates in space, while simultaneously fostering mathematical reasoning and mathematical communication.

Implementation (Step-by-step):

Step 1. Assigning model-solving tasks with increasing levels of complexity.

The teacher designs tasks following a progression from simple to more complex, such as determining vector coordinates, computing vector sums and differences, examining conditions of parallelism or perpendicularity, and calculating lengths and angles. This progression ensures that students can build on the previously constructed models.

Step 2. Organizing problem-solving activities (individual or group work).

Students select appropriate solution strategies, carry out algebraic transformations and calculations, and clearly articulate their reasoning, including the justification for using specific formulas or properties.

Step 3. Providing support through guiding questions.

During students' work, the teacher poses guiding questions such as: “Which quantity needs to be determined?”, “Which vector can represent this relationship?”, “Why is this coordinate system advantageous?”, or “When should the dot product be applied?”. These prompts help students self-regulate and refine their problem-solving approaches.

Step 4. Presentation, discussion, and peer critique.

Group representatives present their solutions, while other groups ask questions or propose alternative approaches. The teacher synthesizes students' ideas, clarifies misconceptions, and emphasizes key methodological points.

Step 5. Generalization and consolidation.

The teacher encourages students to extract general rules or procedures from specific solutions, thereby systematizing problem-solving steps and preparing students to apply these methods to similar situations.

3.3.3. Measure 3. Organizing Learning Activities for Interpreting, Evaluating Results, and Adjusting Mathematical Models in Real-Life Contexts

Purpose:

This measure aims to develop students' ability to interpret mathematical results, evaluate the reasonableness of solutions, and adjust mathematical models, thereby completing the mathematical modeling cycle.

Implementation:

Step 1. Interpreting results in context.

Students translate mathematical results (such as coordinates, lengths, angles, or conditions) into their real-life or geometric meanings, including positions, directions of motion, distances, or spatial relationships.

Step 2. Checking the reasonableness of results.

Students compare the obtained results with intuition, geometric representations, or real-world constraints to verify their validity. This may involve checking units, signs and directions, substituting results back into the model, comparing different solution methods, or using geometric visualization.

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Step 3. Evaluating the model and identifying limitations.

Students identify the assumptions used in constructing the model and analyze their effects on the results. They reflect on aspects in which the model has been simplified and discuss potential sources of inaccuracy or limitations.

Step 4. Adjusting or extending the model.

The teacher encourages students to modify certain conditions (e.g., changing the coordinate system, adding constraints, or altering given data) and predict the impact of these changes. Based on this analysis, students propose adjusted or more generalized models.

Step 5. Concluding the modeling cycle through reflection.

Students summarize the steps they have completed in the mathematical modeling process and conduct self-assessment of their task performance using criteria or observation checklists. This reflection helps consolidate their understanding of modeling as a coherent and iterative process.

IV. CONCLUSIONS

In the context of renewing mathematics teaching toward a competency-based approach, giving priority to the formation and development of mathematical modeling competency for upper secondary school students is an inevitable requirement. This paper has clarified the theoretical foundations of mathematical modeling competency, identified its manifestations among upper secondary school students, and analyzed the potential of the topic *Vector Coordinates in Space* for organizing modeling-oriented learning activities.

On this basis, the study has proposed several pedagogical measures for teaching the topic *Vector Coordinates in Space* toward developing mathematical modeling competency for Grade 12 students. These measures are structured in accordance with the mathematical modeling process, ranging from organizing real-life situations, guiding students to construct and solve mathematical models, to interpreting, evaluating, and adjusting models in real-world contexts. The proposed measures not only promote students' activeness and autonomy in learning but also provide teachers with practical references for designing and implementing competency-oriented mathematics instruction.

The research findings have practical significance in supporting upper secondary mathematics teachers in innovating teaching methods and improving the effectiveness of teaching the topic *Vector Coordinates in Space* in particular, as well as mathematics teaching oriented toward developing mathematical modeling competency in general. Future studies should focus on conducting pedagogical experiments to further examine the feasibility and effectiveness of the proposed measures in different teaching contexts.

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