International Journal of Social Science and Human Research

ISSN (print): 2644-0679, ISSN (online): 2644-0695

Volume 09 Issue 11 November 2026

DOI: 10.47191/ijsshr/v9-i11-35, Impact factor- 8.361

Page No: 9655-9665

From Mathematics Curriculum Objectives to Classroom Practice: A Four-Domain Framework for Effective Teaching

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ABSTRACT: Mathematics curricula ask teachers to develop more than knowledge and procedural skills. They also expect students to reason, solve problems, think creatively, and value mathematics. Turning these broad aims into everyday classroom practice is difficult, because teachers must decide which ideas to emphasise, how to structure lessons, how to draw out students' thinking, and how to judge understanding. This conceptual paper analyses curriculum documents and research literature and organises curriculum objectives into four connected domains: knowledge and skills, mathematical thinking, problem solving, and attitudes and dispositions. For each domain, the discussion explains its significance and the teaching practices that support its development. The analysis is guided by three principles: continuity across year levels, relevance to students' contexts, and equity through differentiation. The paper argues that the four domains reinforce one another and need to be taught together. This requires conceptually focused instruction, mathematical discourse, challenging yet accessible tasks, responsive assessment, and careful use of digital technology and artificial intelligence. The resulting framework gives teachers a practical way to align curriculum intentions with classroom practice and to support students' mathematical proficiency and engagement.

KEYWORDS: mathematics education; curriculum objectives; classroom practice; mathematical thinking; problem solving; attitudes and dispositions

1. INTRODUCTION

Mathematics is a discipline concerned with numerical relationships, patterns, structures, and spatial and geometric forms ([1]). In an era of rapidly advancing digital technologies, it is increasingly applied in science, industry, technology, and everyday life. Mathematics education, however, involves more than developing mathematical knowledge and procedural skills for daily life and further study. It also aims to cultivate students' mathematical reasoning, enhance their problem-solving proficiency, promote creativity, and develop positive dispositions towards mathematics. Mathematics is therefore an essential part of education, supporting students' intellectual development and preparing them to participate effectively in an increasingly complex, technology-oriented society.

Contemporary mathematics curricula emphasise conceptual understanding, procedural fluency, reasoning, problem solving, mathematical communication, and positive dispositions towards mathematics ([2]). However, a persistent challenge lies in translating these broad curriculum objectives into coherent classroom practice. Teachers do not simply implement curriculum resources as prescribed; instead, they interpret broad curriculum expectations, translate them into specific learning goals, design appropriate learning experiences, and adapt curriculum resources based on their professional judgements and classroom contexts ([3,4]). This process involves complex professional judgements about content selection, task design, classroom interactions, student participation, differentiation, and assessment ([2,5]). The effectiveness of curriculum resources therefore depends not only on the quality of the tasks themselves but also on how teachers interpret, adapt, and implement them to support students' mathematical understanding and learning opportunities ([5]).

Research on mathematical tasks highlights the importance of both task design and teachers' classroom enactment of tasks in supporting students' mathematical learning. More broadly, Ahl et al. ([6]), in their systematic mapping review of implementation research in mathematics education, identify limitations in how implementation processes are investigated and reported, particularly regarding planning and design. Their findings highlight the need to examine more closely how research-based ideas are translated into classroom practice. Complementing this perspective, Koichu et al. ([7]) examine the conceptual scope and identity of implementation-related research in mathematics education, positioning it as an important field of inquiry in its own right. Taken together, these studies provide a rationale for examining how broad mathematics curriculum objectives are interpreted and enacted

through pedagogical decisions and learning experiences. However, further conceptual work is needed to clarify these relationships within a coherent framework for curriculum enactment in the classroom.

In response to this gap, this paper develops a conceptual framework that organises curriculum objectives into four interconnected domains: knowledge and skills, mathematical thinking, problem solving, and attitudes and dispositions. These domains build on Kilpatrick et al.'s ([8]) strands of mathematical proficiency but organise them as curriculum objectives linked to classroom practice. The framework was developed in three steps. First, the authors reviewed the mathematics curriculum and teaching documents of [ACARA]([1]), the Victorian Curriculum and Assessment Authority [VCAA]([9]), National Council of Teachers of Mathematics [NCTM]([2]), and Australian Association of Mathematics Teachers [AAMT]([10]) to identify the requirements and standards they set for mathematics learning. Second, drawing on their teaching experience, the authors grouped these expectations into the four domains and generalised them in Table 1. Third, practising mathematics teachers reviewed Table 1 and gave feedback on the domains and their descriptions, which informed the final version. The following sections then used the research literature to examine each domain and the teaching practices that support it. By connecting curriculum expectations with insights from mathematics education research, the paper offers a framework to guide the design of meaningful and coherent classroom learning experiences.

2. LITERATURE REVIEW AND CONCEPTUAL ANALYSIS

2.1 Principles Underpinning Effective Mathematics Curriculum and Teaching

Effective mathematics curriculum and teaching are underpinned by three interconnected principles: continuity of year-level content, relevance to students' contexts, and equity through differentiation. Together, these principles support meaningful, inclusive, and sustained learning.

First, a well-designed curriculum should be grounded in coherently sequenced, year-level content that connects within and across year levels, ensuring continuity in teaching and learning ([1,11]). Teachers need to understand how concepts at each year level build on students' prior knowledge and prepare them for future learning, so that mathematical ideas are developed progressively rather than taught in isolation. Continuity also lets students develop conceptual understanding and procedural fluency together, so they know why a procedure works as well as how to use it ([1,2]).

Second, teaching should be relevant to students by connecting mathematical ideas with personal, social, disciplinary, and real-world situations ([12]). Exploring current real-world issues, such as environmental change, public health, personal finance, or sport, allows students to see mathematics as a tool for understanding the world around them. Using real data for analysis and modelling can strengthen students' problem-solving and reasoning skills. It also engages them as active learners who explore ideas, formulate conjectures, and reflect on their thinking. However, learning contexts should be selected thoughtfully to reflect students' diverse experiences, backgrounds, and identities ([13]).

Third, equity and differentiation are essential to ensure that all students, regardless of their backgrounds, prior achievement, or learning needs, have access to rich mathematical learning and appropriate support ([14, 15]). Differentiation should operate across content, pedagogy, and assessment. In terms of content, teachers can provide multiple entry points and tasks with varying levels of challenge, including scaffolding for students who need support and extension for those ready for greater complexity. In terms of pedagogy, teachers can use flexible grouping, multiple representations, and targeted instruction to respond to students' diverse needs. In terms of assessment, teachers can offer students varied ways to demonstrate their understanding, such as written, oral, and visual responses, so that assessment is fair and informs subsequent teaching decisions ([10,16]).

The following subsections examine how these principles are enacted in practice through curriculum design and teaching approaches (Section 2.1.1), assessment practices (Section 2.1.2), and the purposeful use of digital technology and artificial intelligence (Section 2.1.3).

2.1.1 Curriculum Design and Effective Teaching Approaches

Developmentally appropriate content. A well-designed mathematics curriculum should provide developmentally appropriate content while responding to students' diverse abilities, prior knowledge, experiences, and stages of learning. It should offer all students equitable opportunities to deepen their conceptual understanding, develop procedural fluency, and improve their mathematical achievement. Curriculum content should also respond to contemporary social and technological developments and prepare students to use mathematics in further study, employment, civic participation, and everyday life. At the classroom level, teachers play a critical role in interpreting curriculum objectives, organising mathematical content, selecting instructional strategies, and designing learning experiences that respond to students' prior knowledge and learning needs.

Connecting concrete experiences and representations. Curriculum and instruction should also support students in moving from concrete experiences to increasingly abstract mathematical representations. Practical investigations, experiments, measurements, manipulatives, and mathematical modelling can provide foundations for conceptual development. Teachers should explicitly support students in connecting these experiences with a range of representations, including symbols, diagrams, graphs, tables, formulas, written explanations, and worked examples ([17]). Multiple representations enhance conceptual understanding when students not only interpret and use each one but also explain the relationships among them. Through making these connections, students develop deeper understanding and greater flexibility in their thinking.

Valuing mathematical processes. Mathematics education should not focus exclusively on producing correct answers. It should also emphasise the processes through which solutions are developed, including selecting strategies, using representations, justifying arguments, and evaluating the reasonableness of conclusions. Examining how students reason through problems can reveal conceptual understanding and strategies that may not be apparent from final answers alone ([17]). Both the quality of students' reasoning and the accuracy of their conclusions should therefore be treated as important aspects of mathematical learning and assessment.

Using meaningful contexts. Contextualisation should serve a clear mathematical purpose. Real-world contexts should not distract students from the underlying concepts or create unnecessary linguistic and cognitive demands ([18]). Teachers should therefore select contexts that are accessible, relevant, mathematically meaningful, and appropriate to the intended learning objective.

Balancing student participation and teacher guidance. Teaching is a dynamic and interactive process in which teachers and students engage in reciprocal interactions that shape and support learning. Students should be positioned as active participants who construct and refine mathematical understanding through reasoning, inquiry, discussion, and problem solving ([19,2]). The teacher's role remains essential in setting learning goals, explaining important ideas, selecting tasks, posing purposeful questions, and monitoring progress ([10,2]). Student-centred teaching should therefore not be understood as the withdrawal of teacher direction. Instead, teachers need to structure learning experiences, anticipate students' responses, provide appropriate scaffolding, and offer explicit explanation or modelling when students need additional support ([20,21]). Effective teaching requires a balance between explicit instruction, guided exploration, collaborative learning, and independent activity, determined by the learning goals, the nature of the content, and students' prior knowledge and readiness ([10,22]).

Mathematical discourse. Classroom interaction plays an important role in developing mathematical understanding. Students need opportunities to explain their thinking, compare different strategies, justify their conclusions, and consider the reasoning of others. Discourse enables students to articulate their reasoning and gives teachers valuable evidence of their understanding and misconceptions. Teachers can use this evidence to decide what to do next. They may provide further explanation, pose probing questions, introduce alternative representations, address misconceptions, or extend the challenge. Students also require appropriate linguistic and instructional support to participate meaningfully in mathematical discussions ([23]).

Responsive teaching and differentiation. Responsive teaching should stimulate students' interest, encourage active participation, and support the development of reasoning and creative thinking. Students should have opportunities to observe patterns, explore relationships, ask questions, make conjectures, estimate results, test hypotheses, and reflect on the reasonableness of their conclusions. They should also be encouraged to experiment with different strategies and learn from approaches that do not initially succeed. At the same time, teaching should respond to differences in students' readiness, prior achievement, confidence, and learning needs. Teachers may differentiate by varying representations, prompts, levels of scaffolding, grouping arrangements, and opportunities for extension.

Balancing challenge and support. Teachers therefore need to balance appropriate challenge with suitable support. Excessive assistance may reduce a rich task to a sequence of routine steps, while insufficient support may lead to disengagement and unproductive frustration. Effective support can include clarifying the task, asking focused questions, or encouraging students to consider a different representation. With such support, students can develop mathematical knowledge and skills, understand and apply mathematical ways of thinking, and accumulate meaningful experience of mathematical activity ([17,23]).

2.1.2 Assessment for Mathematics Learning

Assessment in mathematics education serves multiple and interconnected purposes. It should identify students' current understanding, monitor learning progress, support achievement, provide feedback, and inform improvements in teaching. A comprehensive approach should use multiple sources of evidence rather than relying exclusively on formal tests or final answers. Teachers can draw on a range of evidence, including written work, oral explanations, diagrams, calculations, problem-solving strategies, participation in class, and students' own self-assessments and reflections.

Assessment should evaluate not only students' mathematical knowledge and skills but also their ability to communicate their reasoning, select and apply appropriate strategies, and respond to mathematical challenges. Students' confidence, attitudes, and levels of engagement offer valuable insights into how they experience and participate in mathematics learning. Actionable, specific, and timely feedback enables students to recognise their strengths, identify areas for improvement, and determine appropriate next steps ([24]). By engaging with feedback and reflecting on their progress, students can develop the capacity to monitor their own understanding, adjust their learning strategies, and take increasing responsibility for their learning.

Rather than serving only as a measure of achievement after instruction, formative assessment should be embedded within the teaching and learning process. Teachers can use questioning, observation, exit tasks, and classroom discussions to identify students' current understanding and decide what to teach next ([25,26]). Assessment evidence can also help teachers evaluate whether tasks, explanations, examples, and representations are supporting the intended learning.

However, formative assessment does not automatically improve mathematical learning. Its effects depend on how clearly it is defined, how consistently it is implemented, and how effectively teachers respond to the resulting evidence. Recent research suggests that successful implementation requires content-specific learning intentions, appropriate diagnostic processes, frequent and

accessible feedback, consistent use over time, and teachers' capacity to modify subsequent instruction ([27,28,29]). Formative assessment should therefore be understood as a responsive instructional process rather than simply a collection of assessment activities.

2.1.3 Digital Technology and Artificial Intelligence

Advances in digital technology have significant implications for mathematics curriculum design and classroom practice. The integration of technology into mathematics teaching should be guided by mathematical content and pedagogical goals, rather than viewed as an additional or separate component of instruction ([24]). Teachers need to consider when, why, and how a digital tool can support the intended mathematical learning. For example, a graphing tool might be used to connect algebraic and graphical representations, or dynamic software to test a conjecture about shapes.

Digital technologies can provide access to dynamic representations, interactive learning environments, simulations, graphing tools, data sets, and immediate feedback ([30]). These affordances can create opportunities for mathematical exploration that static resources alone may not provide. However, access to technology does not guarantee conceptual understanding. Teachers should help students interpret the outputs of digital tools, explain the mathematical relationships they show, and link them to the underlying concepts and procedures, so that students develop an understanding of the mathematics rather than simply reading off results.

More recently, artificial intelligence (AI) has introduced further opportunities and challenges for mathematics education. Students should be guided to use AI critically, responsibly, and transparently rather than treating it merely as a tool for obtaining answers ([24]). Students should examine AI-generated methods carefully, check the calculations, identify errors or unsupported assumptions, and compare different solution strategies. They should also be able to explain the mathematical reasoning behind a solution in their own words. When aligned with clear curriculum purposes and appropriate teacher guidance, technology-enhanced approaches may support motivation, feedback, independent inquiry, and the development of mathematical and digital competencies ([24]).

2.2 Conceptual Framework: Four Domains of Mathematics Curriculum Objectives

Drawing on curriculum documents and mathematics education research, this paper organises curriculum objectives into four interconnected domains: knowledge and skills, mathematical thinking, problem solving, and attitudes and dispositions. Table 1 summarises the main objective of each domain and its implications for teaching practice. Together, these domains provide the conceptual framework for examining how curriculum objectives can be translated into classroom practice.

Table 1. Four Domains of Mathematics Curriculum Objectives and Their Implications for Classroom Practice

Domain	Main objective	Implications for teaching practice
Knowledge and skills	Students develop conceptual understanding and procedural fluency and apply mathematical knowledge accurately and flexibly.	Teachers connect concepts with procedures, explain underlying principles, use multiple representations, and provide opportunities for purposeful practice and application.
Mathematical thinking	Students identify patterns, make connections, use representations, classify mathematical objects, formulate conjectures, generalise relationships, and justify conclusions.	Teachers design exploratory activities, pose purposeful questions, encourage comparison and generalisation, and facilitate mathematical explanation and argumentation.
Problem solving	Students interpret and analyse problems, select appropriate strategies, apply mathematical methods, evaluate results, and reflect on solutions.	Teachers use meaningful and appropriately challenging problems, maintain cognitive demand, provide responsive support, and encourage comparison and evaluation of solution strategies.
Attitudes and dispositions	Students develop interest, confidence, curiosity, persistence, responsibility, and productive mathematical learning habits.	Teachers create supportive and inclusive environments, establish high but achievable expectations, value students' reasoning, and treat mistakes and revision as parts of mathematical learning.

The four domains build on, and reorganise, the five strands of mathematical proficiency described by Kilpatrick et al.([8]). Conceptual understanding and procedural fluency are combined in knowledge and skills, strategic competence broadly corresponds to problem solving, adaptive reasoning to mathematical thinking, and productive disposition to attitudes and dispositions. The

framework in this paper differs in that it presents these strands as curriculum objectives and links each domain to teaching practices (Table 1) and to the principles of continuity, relevance and equity (Section 2.1).

Although the four domains are analytically distinguishable, they are closely interconnected in classroom learning. Teachers should therefore consider the four domains collectively when planning lessons, designing learning experiences, specifying intended outcomes, and assessing students' learning. Development across the domains provides evidence of meaningful mathematical learning and contributes to students' capacity for continued learning.

2.2.1 Knowledge and Skills

Knowledge and skills form the foundation of mathematics learning and constitute a central objective of the mathematics curriculum. Mathematical knowledge includes understanding concepts, principles, and relationships, while mathematical skills involve the accurate, efficient, and flexible use of procedures, representations, and tools. These two dimensions should not be treated separately. Understanding the principles behind mathematical procedures helps students develop procedural fluency, while using and comparing different procedures can further strengthen their conceptual understanding ([31,32]).

In addition to foundational knowledge and essential skills, intentional mathematics teaching should emphasise the meanings and connections underlying mathematical ideas. Carefully sequenced learning experiences enable students to connect new ideas with prior knowledge, consolidate and extend their understanding, and develop a coherent structure of mathematical knowledge ([1,2]). For example, when teaching algebraic equations, teachers can guide students to explore the properties of equality and explain why particular solution procedures are valid ([33]). Comparing different solution methods allows students to identify shared mathematical principles and determine whether each transformation preserves equality.

To translate this understanding into effective mathematical practice, teachers need to provide opportunities for students to consolidate essential procedures and apply their knowledge flexibly in familiar and unfamiliar situations. One approach is to use a gradual-release structure that moves flexibly between explicit modelling, guided and collaborative practice, and independent application. Often described as "I do, we do, you do," this approach allows teachers to adjust the level of support in response to students' understanding and the demands of the mathematical task. Knowledge and skills also provide an essential foundation for mathematical thinking and problem solving ([8,2]). However, mastery involves more than speed and accuracy; students should also be able to select appropriate procedures, apply them efficiently and flexibly, explain the mathematical principles behind them, and judge whether their answers are reasonable ([1,8]).

2.2.2 Mathematical Thinking

Mathematical thinking is embedded in the construction, development, and application of mathematical knowledge. It includes processes such as abstraction, classification, comparison, generalisation, representation, deductive reasoning, justification, and mathematical modelling. Through these processes, students move beyond the reproduction of facts and procedures to examine mathematical structures, relationships, and underlying principles.

Mathematical thinking develops when students reflect, explore, discuss, and collaborate on mathematically meaningful tasks. To build it, students need chances to recognise patterns, formulate and test conjectures, and generalise relationships, and then to construct arguments that justify their conclusions. Teachers play an essential role by selecting appropriate tasks and posing purposeful questions ([34]). Mathematical thinking also includes metacognition, which is students' capacity to regulate their own thinking. Students use metacognition when they plan an approach, monitor their progress, evaluate whether a strategy is working, and revise a method that is not succeeding. Recent evidence indicates that metacognitive instruction can support mathematical achievement by encouraging students to plan, monitor, evaluate, and revise their approaches to mathematical tasks ([35,36]).

For example, when studying functions, students may compare linear, quadratic, and exponential functions by examining their graphical, tabular, and algebraic representations. They may identify similarities and differences, classify the functions according to their defining characteristics, and generalise patterns concerning their rates of change and long-term behaviour. Teachers can then ask students to explain how features in one representation correspond with features in another. Such activities develop classification, representation, connection, and generalisation as important forms of mathematical thinking. Mathematical thinking enables students to apply mathematical ideas with understanding rather than relying exclusively on memorised procedures.

2.2.3 Problem Solving

Problem solving is both a central objective of mathematics education and an important means through which students apply, integrate, and extend their mathematical knowledge. The process begins with interpreting the problem and identifying relevant information. Students then select and apply strategies while monitoring their progress, and finish by judging whether the result is reasonable and reflecting on their approach. Through problem solving, students learn to coordinate mathematical knowledge, reasoning, representations, and strategies when faced with situations where a solution method is not immediately clear.

Problem solving is closely related to mathematical thinking, but the two domains have different emphases. Mathematical thinking emphasises abstraction, generalisation, and justification, whereas problem solving emphasises coordinating knowledge and strategies to reach a solution. Mathematical thinking therefore supports problem solving, while problem solving provides a context in which mathematical thinking can be applied and developed. This distinction recognises the close relationship between the two domains while maintaining their conceptual differences.

Productive problem-solving instruction involves more than presenting students with unfamiliar questions. Teachers need to select tasks with an appropriate level of cognitive demand, provide support and encourage students to develop, compare, and justify alternative solution strategies ([37,17,38,39]). Teachers can support students without giving away the solution, for example by clarifying the context, asking a focused question, suggesting a useful representation, or inviting them to reconsider an assumption. Such support should preserve opportunities for decision-making, reasoning, and productive struggle.

Statistical investigations provide one example of how problem solving can be integrated into classroom learning. Teachers can design investigations that engage students in collecting, organising, representing, analysing, interpreting, and communicating data ([18]). For example, students may investigate relationships between variables, compare data distributions, select appropriate graphical or numerical representations, draw evidence-based conclusions, and justify their interpretations. They may also evaluate data reliability, identify possible sources of bias, consider the appropriateness of their representations, and acknowledge the limitations of their conclusions. These activities require students to make decisions, evaluate information, coordinate multiple mathematical processes, and apply their knowledge to meaningful problems. Through these experiences, students can strengthen their analytical and reasoning abilities, expand their repertoire of problem-solving strategies, and build greater confidence and persistence when approaching unfamiliar situations. In this sense, problem solving draws on mathematical thinking while also fostering productive dispositions.

2.2.4 Attitudes and Dispositions

Attitudes and dispositions constitute an important dimension of mathematics learning and influence students' participation, persistence, and long-term engagement with the subject. Positive mathematical dispositions include interest, confidence, and curiosity; perseverance and resilience; and a willingness to take responsibility, think openly and rigorously, and collaborate with others ([8,40,41]). In the classroom, these dispositions are visible when students take intellectual risks, think independently, and keep working on unfamiliar problems. Mathematics teaching should therefore attend to students' affective development as well as their cognitive achievement, creating opportunities for students to develop positive attitudes, take responsibility for their learning, and engage meaningfully with mathematical ideas.

Favourable dispositions towards mathematics develop through experiences in the classroom. Teaching practices that offer meaningful challenge, opportunities to explain ideas, appropriate support, and recognition of students' reasoning may foster interest, self-efficacy, persistence, and a willingness to tackle unfamiliar problems ([42,43]). Teachers can further support these dispositions by establishing high but achievable expectations, treating mistakes as opportunities for analysis, valuing different approaches, and normalising the revision of mathematical thinking ([44]).

Classroom discourse, teacher responsiveness, metacognitive instruction, and formative assessment may also influence how students perceive themselves as mathematics learners. These practices can make students' thinking visible, support reflection, and enable teaching to respond more effectively to students' needs ([35,28,45,46]). However, supportive teaching should not eliminate mathematical challenge. Students need opportunities to experience difficulty, evaluate unsuccessful methods, and persevere with appropriate guidance. For example, during a group statistical investigation, students share responsibilities, discuss findings, and present conclusions, and so learn to respect different perspectives, communicate mathematical ideas, and take responsibility for their contributions. When unexpected results or difficulties arise, students can be encouraged to persevere, reconsider their assumptions, and revise their approaches. Such experiences may strengthen students' confidence, resilience, and sense of responsibility. Attitudes and dispositions should therefore not be viewed as secondary outcomes, because they directly shape how students engage in mathematical learning ([8,47,44]).

2.3 Integrating the Four Domains in Classroom Practice

The four domains can be developed effectively through integrated classroom practices, because mathematically rich tasks often draw on several domains simultaneously. In such tasks, students apply conceptual and procedural knowledge, reason about and justify relationships, evaluate problem-solving approaches, and demonstrate productive persistence in the face of challenge [17]. The extent to which such tasks support learning across the four domains depends on their design, the quality of classroom discourse, the balance between student participation and teacher guidance, the provision of differentiated support, and the use of assessment evidence to inform subsequent instruction.

2.3.1 Interrelationships Among the Four Domains

Development in one domain can support and strengthen development in the others. Conceptual understanding and procedural fluency provide students with the mathematical resources required for reasoning and problem solving. Mathematical thinking enables students to recognise structures, connect representations, formulate conjectures, justify conclusions, and determine why mathematical procedures work. Problem-solving experiences, in turn, provide contexts in which students coordinate and extend their knowledge, reasoning, representations, and strategies. Students' attitudes and dispositions influence whether they participate actively in these experiences, persevere through difficulty, reflect on errors, and accept the challenges associated with unfamiliar mathematical situations ([8,2]).

These relationships can be illustrated by returning to the statistical investigation described in Sections 2.2.3 and 2.2.4. In carrying out such an investigation, students draw on knowledge of statistical concepts and procedures, mathematical thinking about patterns

and representations, and problem-solving decisions about methods and interpretations. They also need productive dispositions such as curiosity, responsibility, collaboration, and persistence. The educational value of the activity therefore lies in more than the accuracy of students' calculations: it also lies in how they make decisions, justify interpretations, evaluate evidence, and respond to difficulties.

Integration does not mean that every lesson must address all four domains equally. A lesson may give greater attention to a particular concept, procedure, reasoning process, or problem-solving strategy. Across a coherent sequence of lessons, however, students should encounter opportunities to connect knowledge with reasoning, apply learning to different situations, communicate mathematical ideas, reflect on their approaches, and develop increasing independence. The four-domain framework can therefore guide long-term planning while remaining responsive to the specific purposes of individual lessons.

2.3.2 Student Participation and Teacher Guidance

Integrating the four domains requires active student participation together with purposeful teacher guidance, as outlined in Section 2.1.1. The balance between participation and guidance should vary with the content, the intended outcomes, and students' prior knowledge and readiness. It should be flexible rather than linear, because teachers may need to return to explanation or modelling when misconceptions emerge.

2.3.3 Tasks and Discourse

Carefully designed tasks provide an important way to integrate the four domains. A good task has a clear mathematical purpose. It gives students reasons to link concepts with procedures, look for patterns, move between representations, and decide on a strategy, and it asks them to explain their thinking. Mathematical discourse strengthens the value of these tasks by making students' thinking visible. Teachers can facilitate productive discussion by anticipating likely approaches, monitoring students' work, selecting and sequencing contributions, and connecting individual strategies to broader mathematical principles ([48]).

2.3.4 Differentiation and Inclusive Practices

Differentiation is necessary to ensure that all students can participate meaningfully in cognitively worthwhile mathematical activity. Differentiated instruction involves proactively adapting content, process, product, and learning environments to support learner diversity while maintaining common learning goals ([49,50]). However, differentiation should not result in some students receiving only routine procedural work while others are given opportunities to reason and solve problems. All students should have access to important mathematical ideas and opportunities to explain, investigate, and make decisions, supported by accessible entry points and high but achievable expectations. Effective differentiation recognises learner diversity as a normal feature of teaching rather than an exception requiring separate provision.

Assessment supports differentiation and responsive teaching by providing evidence of students' learning and instructional needs (see Section 2.1.2). Marks et al. ([51]) emphasise formative assessment as the basis for effective differentiation. Clearly communicated learning intentions and success criteria help students understand what they are learning and what successful performance looks like, while feedback exchanged between teachers, students, and peers supports students in monitoring and regulating their own learning ([25,26]).

2.3.5 Aligning Teaching, Assessment, Learning and Technology

Integrating the four domains requires alignment among learning goals, tasks, classroom discourse, teacher guidance, differentiation, assessment, and the purposeful use of digital technologies and artificial intelligence. This alignment can be viewed as a continuous cycle of responsive instruction (Figure 1).

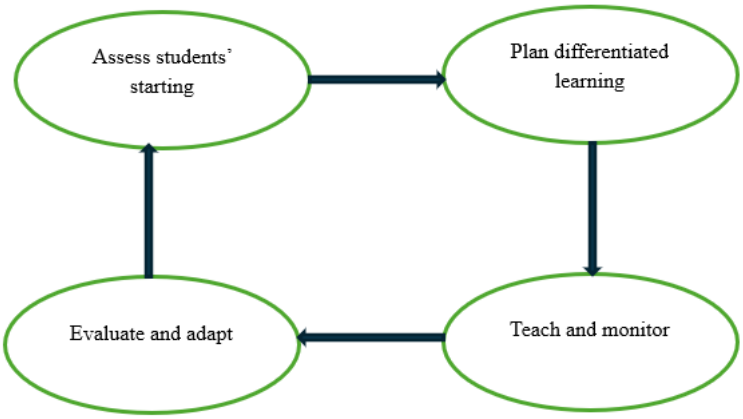


Figure 1. A cyclical model for aligning teaching, assessment, and learning.

The cycle begins with assessing students' starting points and progresses through planning differentiated learning experiences, teaching and monitoring learning through instruction and mathematical discourse, and evaluating evidence to inform subsequent

instruction. The process is iterative and supports responsive teaching that promotes mathematical knowledge, reasoning, problem-solving capabilities, and productive dispositions. Within this cycle, tasks and discourse make students' thinking visible, assessment evidence guides adjustments to instruction, and differentiation keeps meaningful learning accessible to all students. Digital technologies and artificial intelligence can support each stage of the cycle, for example by providing evidence of learning and immediate feedback, but they should complement, rather than replace, mathematical thinking, communication, and problem solving (see Section 2.1.3).

3. IMPLICATIONS FOR MATHEMATICS TEACHING

3.1 Implications for Classroom Practice

The four-domain framework has six implications for classroom teaching. First, learning goals should include mathematical thinking, problem solving, and dispositions as well as knowledge and skills, and should be planned as interconnected dimensions of learning. Second, tasks should be selected for the kinds of thinking, decision-making, communication, and persistence they require, as well as for their mathematical content. Third, teachers should balance explicit guidance with active student participation. Fourth, mathematical discourse should be used to connect the four domains and generate evidence of learning. Fifth, differentiation and assessment should support equitable participation in worthwhile mathematical activity, and assessment should look beyond accuracy to conceptual understanding, strategy selection, communication, persistence, and responsiveness to feedback, so that evidence informs subsequent instruction. Sixth, digital technology and AI should be used purposefully to support exploration, representation, and feedback.

These implications can be illustrated through a lesson sequence on rate of change. Students might first explore a real-world situation involving the relationship between distance and time. They could identify patterns and make predictions, represent the relationship using tables, graphs, and equations, and apply their understanding to solve unfamiliar problems. Through classroom discussion, students would explain and justify their strategies, compare different approaches, and respond to alternative perspectives. Throughout the sequence, teachers could attend to students' confidence, persistence, willingness to take risks, and engagement with mathematical challenges. Such a sequence illustrates how the four domains can be integrated within a coherent learning experience rather than addressed as isolated instructional objectives.

To support the practical application of the framework, teachers might consider the following questions when planning mathematics learning experiences:

- Knowledge and skills: What mathematical knowledge, concepts, and skills should students develop?
- Mathematical thinking and problem solving: What opportunities will students have to reason, investigate, make decisions, and solve problems?
- Attitudes and dispositions: How will the learning experience foster positive mathematical attitudes, confidence, persistence, and intellectual curiosity?
- Integration: How will students communicate, collaborate, evaluate different perspectives, and engage with mathematical ideas?

The framework does not prescribe a single method; it supports context-sensitive decisions about how different approaches can contribute to students' broad mathematical development.

3.2 Implications for Curriculum Design and Teacher Education

For curriculum designers, the framework highlights the importance of developing learning outcomes, content descriptions, and assessment approaches that recognise the interconnected nature of mathematical knowledge, thinking, dispositions, and other dimensions of learning. Rather than prioritising measurable knowledge and skills alone, curriculum frameworks should provide opportunities for students to develop as confident, reflective, and engaged mathematical learners.

For teacher education, the framework highlights the need to prepare teachers to design learning experiences that integrate these dimensions, make informed pedagogical decisions, and recognise diverse forms of evidence of students' mathematical development. This requires teachers to understand not only what students learn in mathematics but also how they engage with, make sense of, and develop relationships with the subject.

4. CONCLUSION

This paper has presented a four-domain framework, comprising knowledge and skills, mathematical thinking, problem solving, and attitudes and dispositions, for translating broad mathematics curriculum objectives into classroom practice. Although analytically distinguishable, the domains are mutually reinforcing. Effective teaching therefore addresses the domains together, through aligned learning goals, well-designed tasks, mathematical discourse, purposeful guidance, differentiation, responsive assessment, and critical use of technology.

The framework offers a curriculum-informed basis for planning, analysing, and evaluating mathematics teaching across contexts, as well as informing curriculum design and teacher education. It seeks to bridge the gap between curriculum aspirations and

classroom practice. Although Table 1 was refined through feedback from practising teachers, the framework remains a conceptual analysis and has not been empirically tested. Further research could examine its application across mathematical topics, year levels, and instructional settings, as well as investigate how different teaching approaches, curriculum designs, and teacher education programs support development within and across the four domains.

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