

Process for Designing Fraction Instructional Activities to Foster Critical Thinking in Grade 4 Students

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ABSTRACT: Developing critical thinking in mathematics instruction requires broad objectives to be translated into specific learning activities appropriate to the content and students' level. This study aims to establish principles and propose a process for designing fraction instructional activities to foster critical thinking in Grade 4 students. The study employs document analysis and synthesis, systematization and generalization, combined with an illustrative case study. Drawing on Facione's framework of six critical-thinking skills and the learning outcomes of the 2018 General Education Curriculum in Mathematics, the study identifies manifestations of critical thinking in fraction learning, five design principles, and a five-step process: identifying learning outcomes and critical-thinking objectives; selecting focal content and cognitive bottlenecks; developing tasks and anticipating responses; designing questions and modes of organization; and reviewing, revising, and finalizing. The process is illustrated through an instructional activity on adding $\frac{1}{3}$ and $\frac{1}{6}$. The analysis indicates that demands for interpretation, analysis, evaluation, explanation, and self-regulation can be increased without extending the content beyond the Grade 4 curriculum. The process provides a design framework; its feasibility and effects need to be further examined through empirical research.

KEYWORDS: critical thinking; fractions; instructional activity design; mathematics instruction; Grade 4 students

1. INTRODUCTION

Developing critical thinking is an important orientation in modern education, in which learners need not only to receive and reproduce knowledge but also to interpret information, analyze relationships, evaluate evidence, draw conclusions, and reconsider their own thinking processes. According to the findings of the Delphi Project led by Facione (1990), critical thinking is understood as purposeful, self-regulatory judgment manifested through six core cognitive skills: interpretation, analysis, evaluation, inference, explanation, and self-regulation. This framework provides a basis for identifying critical thinking through specific cognitive actions, rather than equating critical thinking with refuting other people's views or merely identifying an incorrect answer. However, critical thinking does not develop automatically simply because students participate in a learning activity. Abrami et al.'s (2015) meta-analysis of 341 effect sizes shows that critical thinking can be fostered through instructional interventions, but the magnitude of the effect depends on how instruction is organized; opportunities for dialogue, problems situated in specific contexts, and teacher guidance are factors with positive effects. This creates a need to translate the objective of developing critical thinking into intentionally designed learning activities and tasks.

In mathematics instruction, learning activities enable students to activate knowledge, select approaches, explain reasoning, and evaluate the reasonableness of results. Mathematical tasks are therefore regarded as an important means of creating opportunities for students to think and reason. Stein, Grover, and Henningsen (1996) indicate that the level of thinking students can engage in is substantially influenced by the cognitive demands of a task; at the same time, those demands can be maintained or reduced as the teacher organizes its implementation. Thus, a task with high cognitive potential does not ensure that students will actually analyze, evaluate, or explain if the instructional activity is directed only toward obtaining the correct answer quickly. The study by Kazemi and Stipek (2001), conducted in upper-elementary classrooms working on fraction addition, shows that environments promoting conceptual thinking have such characteristics as: explanations must contain mathematical reasoning rather than merely describe procedures; students examine relationships among multiple strategies; errors are used as opportunities to reanalyze the problem; and consensus must be established through mathematical reasoning. These findings show that designing instructional activities to foster critical thinking requires simultaneous attention to mathematical tasks, the teacher's system of questions, and the ways in which students are organized to present, examine, defend, or revise their views.

Fractions constitute a content domain well suited to organizing such activities, but they also pose many cognitive challenges for students. Unlike whole numbers, the value of a fraction is determined by the relationship between its numerator and denominator;

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therefore, reasoning that is valid for whole numbers cannot always be transferred directly to fractions. Ni and Zhou (2005) use the concept of “whole number bias” to analyze learners’ tendency to apply characteristics or rules of whole numbers when working with fractions and rational numbers. This bias may be manifested when students believe that a fraction with a larger denominator has a greater value, or when they add or subtract fractions by operating independently on the numerators and denominators. Siegler and Lortie-Forgues (2017) likewise argue that difficulties in rational-number arithmetic arise from both the inherent characteristics of the rational-number system and limitations in instruction. The significance of developing a sound understanding of fractions is also demonstrated in the longitudinal study by Siegler et al. (2012): knowledge of fractions and division in elementary school independently predicted algebra knowledge and mathematics achievement in high school, even after controlling for several cognitive factors, other mathematical knowledge, and family characteristics. Fraction instruction should therefore not focus solely on memorizing and applying rules, but should enable students to explain meanings, analyze relationships, examine reasonableness, and identify the causes of errors.

In the Vietnamese context, the 2018 General Education Curriculum in Mathematics identifies mathematical thinking and reasoning competency as a core component of mathematical competency. At the primary level, students are expected to perform simple thinking operations, observe and identify similarities and differences, state the basis for a solution method, and answer questions when reasoning and solving problems (Ministry of Education and Training, 2018). For Grade 4 fraction content, the curriculum sets out learning outcomes such as recognizing the initial concept and basic property of fractions; simplifying fractions and bringing them to a common denominator in simple cases; comparing and ordering fractions in the specified cases; adding and subtracting two fractions with the same denominator or with unlike denominators when one denominator is divisible by the other; and solving several related real-world problems. These outcomes make it possible to organize activities in which students not only perform calculations but also interpret given information, select a basis for comparison, analyze solution methods, evaluate results, and correct errors. For example, when adding two fractions with unlike denominators in the case where one denominator is divisible by the other, students may be asked to examine two solution methods, identify the reasonable method, and explain why the fractions need to be expressed in the same fractional unit before addition is performed. This content lies entirely within the learning outcomes of the Grade 4 curriculum.

Although previous studies have provided important foundations concerning the structure of critical thinking, difficulties in learning fractions, and the role of mathematical tasks in thinking activity, a gap remains between theoretical orientations and the design of specific instructional activities. Within the body of literature reviewed in this study, studies commonly focus on one of three directions: identifying components of or strategies for developing critical thinking; analyzing students’ understanding of and errors concerning fractions; or examining the cognitive demands of mathematical tasks. There are still few guidelines that specify how teachers can translate the learning outcomes of Grade 4 fraction content and the components of critical thinking into a process for designing instructional activities that can be applied directly. This gap is particularly noteworthy in the context of the Vietnamese curriculum, where designed activities must both create opportunities for students to analyze, evaluate, and self-regulate and strictly adhere to the scope of knowledge and level of demand for Grade 4 students.

Based on the foregoing analysis, this study aims to propose principles and a process for designing fraction instructional activities to foster critical thinking in Grade 4 students, while illustrating the application of the process through a case involving the addition of two fractions with unlike denominators. In this study, the phrase “to foster critical thinking” denotes a design orientation that creates opportunities for students to engage in actions such as interpretation, analysis, evaluation, explanation, and self-regulation; it does not imply that the study has measured or demonstrated the process’s effect on the development of students’ critical thinking.

The study focuses on answering two questions:

1. How should theoretical requirements and the requirements of Grade 4 fraction content be translated into a basis for designing instructional activities to foster critical thinking?
2. What principles need to be ensured and what process should be implemented when designing fraction instructional activities to foster critical thinking in Grade 4 students, and how is that process manifested through a specific illustrative case?

2. RESEARCH METHODS

The study employs the following methods:

- Document analysis and synthesis: used to analyze studies on critical thinking, fraction instruction, and mathematical task design, and to compare them with the learning outcomes for Grade 4 fraction content in the 2018 General Education Curriculum in Mathematics.
- Systematization and generalization: used to establish principles and develop a process for designing fraction instructional activities that create opportunities for students to interpret, analyze, evaluate, explain, and revise their solution methods.
- Illustrative case study: used to analyze Example 2.2 on teaching the addition of two fractions with unlike denominators when one denominator is divisible by the other, thereby clarifying how the steps of the proposed process are applied.

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The methods above serve the theoretical development and illustration of the process; the study does not collect empirical data and does not aim to test the process's effects on students.

3. RESEARCH RESULTS

3.1. Critical Thinking in Grade 4 Fraction Learning

According to Facione (1990), critical thinking is a process of making purposeful, self-regulatory judgments, manifested through six cognitive skills: interpretation, analysis, evaluation, inference, explanation, and self-regulation. However, these skills are meaningful for instructional design only when they are translated into actions associated with specific mathematical content. In this study, critical thinking in fraction learning is understood as the process in which students use mathematical knowledge, representations, and reasoning to understand a task; analyze relationships among given data; consider the correctness or reasonableness of statements and solution methods; draw and explain conclusions; and examine and revise their thinking when more appropriate evidence emerges. Accordingly, a correct answer is not sufficient to reflect critical thinking; the evidence and the process through which students reach the answer must also be considered.

Table 1. Manifestations of Critical-Thinking Skills in Fraction Learning

Skill	Manifestation in Grade 4 fraction learning
Interpretation	State the meaning of a fraction.
Analysis	Examine the relationship between the numerator and denominator, between the object being acted upon, the part, and the whole.
Evaluation	Judge the correctness and reasonableness of a result, solution, or statement on the basis of the properties and mathematical representations of fractions.
Inference	Use given information and previously learned knowledge to make predictions, select a solution method, or draw conclusions about issues related to fractions and operations on fractions.
Explanation	Present a solution method and state the basis for a conclusion using words, symbols, or models of fractions and operations on fractions.
Self-regulation	Review the implementation process, detect errors, and revise the solution method on the basis of mathematical evidence.

The manifestations in Table 1 are used to guide activity design. An activity also does not necessarily need to engage all six skills simultaneously. Depending on the objective and lesson content, the teacher may select several focal manifestations, but the task needs to create opportunities for students to undertake at least one action involving examination of evidence, evaluation, explanation, or revision, rather than merely reproducing a rule.

3.2. Grade 4 Fraction Content and the Potential for Creating Critical-Thinking Activities

According to the 2018 General Education Curriculum in Mathematics, Grade 4 fraction content comprises four main topics: the initial concept of fractions; the basic property of fractions; comparison of fractions; and addition, subtraction, multiplication, and division of fractions (Ministry of Education and Training, 2018).

Regarding the basic property, students simplify fractions in simple cases and bring two fractions to a common denominator when one denominator is divisible by the other. The comparison and ordering of fractions are limited to fractions with the same denominator or fractions for which one denominator is divisible by the remaining denominator or denominators. For operations, students perform addition and subtraction in analogous cases; multiply and divide two fractions; and solve several problems involving up to two or three computational steps with the four operations on fractions.

These contents create several cognitive bottlenecks that can be used to design activities to foster critical thinking:

- When recognizing and representing fractions, students need to identify the unit whole correctly and recognize that the parts must be equal.
- When applying the basic property, students need to explain why multiplying or dividing both the numerator and denominator by the same nonzero number does not change the value of the fraction.
- When comparing fractions, students may be influenced by whole-number thinking and compare only the numerators or only the denominators.
- When adding or subtracting fractions with unlike denominators, students may add the numerators directly and the denominators directly, or mechanically bring the fractions to a common denominator without understanding the need to express them in the same fractional unit.

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The final difficulty is related to the phenomenon whereby learners apply whole-number thinking to fractions, as analyzed by Ni and Zhou (2005). Operations with fractions may also become difficult to understand when students merely memorize rules without establishing relationships between computational procedures and the meaning of fractions (Siegler & Lortie-Forgues, 2017).

However, fraction content in itself does not automatically generate critical thinking. Even an exercise that concerns a knowledge point prone to misunderstanding still primarily assesses procedural knowledge if it only requires students to apply a formula. The potential to develop thinking emerges only when the task places students in a situation where they need to interpret, select, verify, explain, or revise a solution method.

3.3. Principles for Designing Fraction Instructional Activities to Foster Critical Thinking

Based on a comparison of critical-thinking theory, the characteristics of fraction learning, and curriculum requirements, the study establishes five design principles.

Principle 1. Adhere to learning outcomes and ensure mathematical accuracy

Each activity must originate from a specific learning outcome in the Grade 4 curriculum. Task difficulty must not be created by introducing content from higher grades, but by increasing the demands placed on cognitive activity, such as analysis, evaluation, explanation, and self-checking. Contexts and visual models must accurately represent the unit whole, the equality of the parts, and relationships among fractions.

Principle 2. Identify both mathematical objectives and manifestations of critical thinking

In addition to objectives concerning mathematical knowledge and skills, the teacher needs to determine what judgment students will make, what evidence they will use, and what learning product they will produce. For example, the objective may be not only to add two fractions with unlike denominators, but also to explain why the two fractions must be brought to a common denominator or to evaluate an inaccurate addition method.

Principle 3. Create a need to examine and make evidence-based judgments

The task needs to contain an element that requires students to think and choose, such as a difference between two solutions, a potentially debatable statement, a typical error, or an inconsistency between a computational result and a visual model. However, not every task necessarily needs to have a real-world context or multiple solution methods. The important consideration is that students have a specific object to analyze and mathematical evidence on which to base a judgment.

Principle 4. Ensure an appropriate level of difficulty and provide suitable support

Questions need to be concise, clearly state the task, and be appropriate to Grade 4 students' reading comprehension. Models, manipulatives, and prompting questions are used to support students' access to the problem, after which the level of support should be gradually reduced so that students explain and check their solution methods independently. The use of images is meaningful only when the images are accurate and are linked to questions that require students to extract information from them.

Principle 5. Ensure coherence among the task, activity organization, and evidence of learning

The mathematical task is the core, but opportunities for thinking also depend on how the teacher poses questions and organizes discussion. The teacher needs to anticipate different responses, enable students to present and comment on the basis of reasoning, and conclude the activity by formalizing the knowledge. Evidence of learning needs to include explanations, the basis for choices, or traces of revision, rather than only the final answer. This view is consistent with research findings on the role of cognitive demand in mathematical tasks and of classroom discourse oriented toward evidence-based reasoning (Stein et al., 1996; Kazemi & Stipek, 2001).

3.4. Process for Designing Fraction Instructional Activities

Based on the principles above, the study proposes a five-step process:



In Step 1, the thinking objective needs to be expressed as an observable action. General formulations such as “develop critical thinking” are insufficient to guide design. The teacher needs to specify what students will analyze, which statement they will evaluate, what evidence they will use to explain, or at which step they will self-regulate.

In Step 2, the teacher identifies the “bottleneck” in the content. This may be a relationship that is difficult to recognize, a common misconception, or a solution step that can easily be performed mechanically. The bottleneck must lie within the scope of knowledge students already possess or can discover using the tools provided.

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In Step 3, the task is developed so that the bottleneck becomes a problem students need to solve. Anticipating responses helps the teacher prepare to use different approaches, but does not presume that all students will think in the same way.

In Step 4, questions are sequenced to guide students from understanding the task and analyzing the given data to evaluating, explaining, and self-checking. A “why” question is genuinely useful only when students have a specific statement, model, or solution method to examine. The teacher also needs to determine when students will work individually, exchange ideas with a peer, and participate in whole-class discussion.

In Step 5, the activity is reviewed using the following questions: Is the content consistent with Grade 4 requirements? Are the given data and images accurate? Do students have to use mathematical evidence? Through what product can the teacher identify the students’ thinking process? Can the activity be implemented within the lesson time? If these conditions are not met, the teacher returns to the earlier steps to make revisions.

3.5. Illustration of Teaching the Addition of Two Fractions with Unlike Denominators to Foster Critical Thinking in Students

The case selected for illustration is an activity for developing the method of adding $\frac{1}{3}$ and $\frac{1}{6}$.

Step 1. Identify learning outcomes and critical-thinking objectives

The learning outcome is to add two fractions with unlike denominators when one denominator is divisible by the other.

In addition to the objective of performing the calculation, the activity aims to create opportunities for students to:

- Interpret the meaning of $\frac{1}{3}$ and $\frac{1}{6}$ in relation to the same unit whole;
- Analyze what is unreasonable about directly adding the numerators and denominators;
- Evaluate the alternatives on the basis of the model and the properties of fractions;
- Explain why the two fractions need to be brought to a common denominator;
- Check and revise the solution method when an error is detected.

Step 2. Select the cognitive bottleneck

The cognitive bottleneck of the activity is the difference in fractional units. One third and one sixth of the same model are not parts of the same size; therefore, they cannot be added by directly counting the numerators and denominators. Students may obtain the result $\frac{2}{9}$ by extending whole-number addition to the numerators and denominators, or they may bring the fractions to a common denominator by applying a rule without being able to explain the meaning of this operation.

Step 3. Develop the task and anticipate responses

The teacher presents the following situation:

Minh is assembling a model house. In the morning, Minh assembles $\frac{1}{3}$ of the model. In the afternoon, Minh assembles another part equal to $\frac{1}{6}$ of the model. What fraction of the model has Minh assembled over the two sessions?

The teacher provides a rectangular-strip model representing the same unit whole and introduces two solution methods:

$$\text{Method 1: } \frac{1}{3} + \frac{1}{6} = \frac{2}{9}.$$

$$\text{Method 2: } \frac{1}{3} = \frac{2}{6}; \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}.$$

The central question of the task is: Can both of the methods above be correct? What evidence do you use to reach your conclusion? Anticipated responses include: students select Method 1 because it adds the numbers in the numerators and denominators; select Method 2 because they remember the rule for bringing fractions to a common denominator; or select Method 2 and use the model to explain that $\frac{1}{3}$ corresponds to $\frac{2}{6}$ of the same whole.

Step 4. Design questions and organize the activity

First, students work individually, select the solution method they consider correct, and record their reason. They then work in pairs and use the visual model to check their choice. The teacher may use the following questions:

- $\frac{1}{3}$ and $\frac{1}{6}$ represent which parts of the same model?
- Are the parts represented by the two fractions the same size?
- How was the result $\frac{2}{9}$ obtained? Has that method taken the meaning of the denominator into account?
- What evidence does the model provide for Method 2?
- If you initially selected Method 1, which step do you need to revise, and why?

During the whole-class discussion, the teacher does not immediately confirm the answer but asks students to present their evidence. After the alternatives have been analyzed, the teacher formalizes the knowledge: to add two fractions with unlike denominators in this case, they need to be brought to a common denominator so that the parts are represented in the same fractional unit; only then are the numerators added.

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At the end of the activity, students briefly answer two questions: “What evidence helped you identify the correct method?” and “Before adding two fractions with unlike denominators, what do you need to check?” This activity is intended to create trace evidence of self-checking and revision without substantially increasing the writing demands placed on students.

Step 5. Review and finalize

The task remains within the limits of Grade 4 content because the addition is performed in the case where one denominator is divisible by the other. The activity does not require students to find the least common multiple or address the general case of two denominators that do not have a divisibility relationship.

When presented, the two figures or strips must be the same size to represent the same unit whole; the parts within each figure must be equal. A house image that is merely decorative or divided into unequal parts should not be used, because such an image cannot serve as mathematical evidence. The incorrect solution $\frac{2}{9}$ must also be analyzed and followed by a clear formalization step, so that students do not simultaneously retain both the correct and incorrect methods without a basis for distinguishing between them.

The case above shows that the five-step process can transform a familiar computational problem into an activity that requires students to interpret, analyze, evaluate, explain, and revise within the prescribed curriculum.

4. DISCUSSION

The research results clarify how the components of critical thinking can be translated from the theoretical level into specific decisions in the design of fraction instructional activities. The five-step process connects learning outcomes, critical-thinking objectives, cognitive bottlenecks, learning tasks, anticipated responses, and modes of organization. This approach is consistent with Facione’s (1990) conception of interpretation, analysis, evaluation, inference, explanation, and self-regulation, while also reflecting the domain-specific nature of critical thinking emphasized by Ennis (1989). In fraction learning, students can evaluate and defend a conclusion only when they know how to use appropriate mathematical evidence, such as the unit whole, the equality of the parts, the basic property of fractions, or the relationship between visual and symbolic representations.

A noteworthy feature of the process is that it increases the level of thinking required without extending the content beyond the curriculum. Example 2.2, concerning the addition of two fractions with unlike denominators when one denominator is divisible by the other, is consistent with the Grade 4 mathematics learning outcomes. The cognitive demand is increased by having students analyze two solution methods, identify what is unreasonable, select evidence, explain the need to bring the fractions to a common denominator, and revise the solution. This shows that developing critical thinking does not necessarily need to be associated with more difficult problems or content from higher grades; the focus lies in the quality of the activity of examination, evaluation, and explanation.

The use of an incorrect method in Example 2.2 is also consistent with the view that errors can serve as a starting point for mathematical inquiry (Borasi, 1994). At the same time, the study by Kazemi and Stipek (2001) shows that explaining through mathematical reasoning, examining relationships among strategies, and using errors are important for conceptual thinking in the mathematics classroom. However, an incorrect solution does not automatically generate critical thinking. If the teacher only asks students to choose correct or incorrect and then announces the answer, the task may still remain at the level of recognition. An error is valuable only when students are asked to identify what is unreasonable, use a model or mathematical property to verify it, explain its cause, and correct the solution method.

Anticipating student responses and designing the mode of organization are important conditions for maintaining the thinking demands of a task. Stein, Grover, and Henningsen (1996) indicate that a task with high cognitive demand can still be simplified during implementation. The teacher therefore needs to refrain from providing a solution too early, allow time for students to present and compare alternatives, and use questions directed toward specific evidence rather than asking only the general question “Why?” Evidence of critical thinking should not be inferred from a correct answer, but needs to be identified in how students reason, verify, respond to other views, and revise their solution methods.

Nevertheless, the proposed process has thus far been developed and illustrated on the basis of theoretical research. The study has not collected empirical data and therefore cannot affirm the process’s feasibility or effects on students. Example 2.2 only clarifies how the steps are applied; it is not evidence of effectiveness. Future studies need to validate the process through the views of experts and teachers, pilot the task with a small group of students, and then conduct classroom research to analyze responses, the level of support required, and the possibility of application to other content in the fraction topic. The study’s principal contribution is therefore to provide a design framework as a basis for further refinement and empirical testing.

5. CONCLUSION

This study proposed a five-step process for designing fraction instructional activities to foster critical thinking among Grade 4 students. Grounded in Facione’s framework and the learning outcomes of the 2018 General Education Curriculum in Mathematics, the process connects the identification of learning outcomes and critical-thinking objectives with the selection of focal content and cognitive bottlenecks, task development and anticipation of student responses, question design and modes of organization, and final

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review and revision. The illustrative case involving the addition of fraction to demonstrate how a familiar curriculum-based task can be redesigned to create opportunities for students to interpret fraction meanings, analyze alternative solution methods, evaluate mathematical evidence, explain conclusions, and revise errors without introducing content beyond Grade 4 requirements. The proposed process therefore provides teachers with a theoretically grounded design framework for coordinating mathematical content, cognitive demand, classroom interaction, and evidence of learning. Nevertheless, the process has been developed and illustrated through theoretical analysis only. The case presented should not be treated as evidence of effectiveness. Further studies should seek expert and teacher validation, pilot the designed activities with small groups of students, and conduct classroom-based empirical research to examine its feasibility, required levels of support, and potential effects on students' critical thinking in fraction learning.

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